Author-created version of the journal article

ON THE ORIGIN OF MASS IN THE STANDARD MODEL

Electronic version of an article published as
International Journal of Modern Physics E
Vol. 22, No. 1 (2013) 1350002 (36 pages)
DOI: 10.1142/S021830131350002X
© World Scientific Publishing Company
www.worldscientific.com

Errata

Section 3, first paragraph, last line: Change $B\alpha + 2B\alpha$ to $B\alpha + 2(B\alpha)^2$.

Section 6.2: In second paragraph below Eq. (44), I state about the electron-type Higgs that "at its first encounter with matter (electrons or quarks), it will most probably transform into a pair of ordinary photons." This conclusion is wrong. See page 23 in http://www.physicsideas.com/Conclusions.pdf.

On the origin of mass in the standard model

Stig Sundman
Dragonvägen 12 A 5, FIN-00330 Helsinki, Finland

2 January 2013

Abstract

A model is proposed in which the presently existing elementary particles are the result of an evolution proceeding from the simplest possible particle state to successively more complex states via a series of symmetry-breaking transitions. The properties of two fossil particles — the tauon and muon — together with the observed photon–baryon number ratio provide information that makes it possible to track the early development of particles. A computer simulation of the evolution reveals details about the purpose and history of all presently known elementary particles. In particular, it is concluded that the heavy Higgs particle that generates the bulk of the mass of the Z and W bosons also comes in a light version, which generates small mass contributions to the charged leptons. The predicted mass of this "flyweight" Higgs boson is 0.505 MeV/c^2, 106.086 eV/c^2, or 12.0007 μeV/c^2 (corresponding to a photon of frequency 2.9018 GHz) depending on whether it is associated with the tauon, muon, or electron. Support for the conclusion comes from the Brookhaven muon $g-2$ experiment, which indicates the existence of a Higgs particle lighter than the muon.

Keywords: Origin of mass; muon–electron mass ratio; JBW finite QED hypothesis; flyweight Higgs boson; origin of particles.

PACS Number(s): 12.10.–g, 98.80.Bp

Contents

1 Introduction and summary

In our standard picture of the world, we take the speed of light (c) and particle lifetimes (τ) to be constant over time. From the law of conservation of energy, it follows that particle rest energy ($E_0 = mc^2$) and mass (m) are constant, too. However, in an expanding universe, the photon energy $E_\gamma = hc/\lambda$ decreases as the wavelength λ increases for photons traversing intergalactic distances. This redshifting of light implies that the universe loses energy.

However, since one should not without good reason abandon the idea of universal applicability of the law of conservation of energy, the fact that the universe loses energy in its standard description may be taken to mean that there exists an alternative description in which its total energy is conserved, but particle rest energy is not. In this "global picture" of the world,

$$Mc^2 + Nhc/\lambda = \text{const.} \tag{1}$$

if M is the mass content of the universe and, in addition to the mass, there are N background photons of average energy $h\nu = hc/\lambda$. Equation (1) implies that the universe's matter energy increases with c^2, while its radiation energy decreases because c increases at a slower rate than the wavelength λ does.

A consequence of global conservation of energy is that, according to Eq. (1), an expanding and purely radiative ($M = 0$) universe is forbidden. This observation explains the matter–antimatter asymmetry of the world. For instance, in a universe consisting of a single proton–antiproton pair in a sea of background photons, annihilation of the massive particle pair would be inhibited. Instead, the proton–antiproton pair would be forced to undergo a symmetry-breaking transition into a proton–electron pair — a kind of nuclear "big bang" that introduces kinetic energy and heats the universe.

A computer simulation of the early phases of the universe convincingly demonstrates that the universe has evolved through a series of symmetry-breaking transitions from the perfectly symmetric state of literally nothing to its present phase. Evidence of this evolution is provided by the existence of two "fossil electrons" (tauon and muon) and by the large ratio (n_γ/n_b) between the observed numbers of background photons and baryons (protons and neutrons).

The simulation reveals several surprising details. The first one is that both c and τ grow over time with $\tau \propto c$. At first sight this result may seem paradoxical. Can a given distance, $d = \tau c$, be constant in one picture and grow in another? The answer is that, in a classical world it cannot, but in a quantum world where distance is undefinable, it can.

From the apparent paradox, it follows that one must distinguish between the "global picture," in which particle lifetimes or atomic-clock ticks increase in length, and our standard, "local picture," in which we measure time by counting clock ticks. In the former picture, the universe is younger than in the latter picture if we use the length of today's second (or atomic-clock tick) as common yardstick in both pictures.

Briefly, the simulation, which is performed in the global picture, shows that the mass-bearing particles have evolved according to

$$\mathrm{D} \rightarrow \tau_0^+\tau_0^- \rightarrow \mu_0^+\mu_0^- \rightarrow \mathrm{e}^+\mathrm{e}^- \rightarrow \pi^+\pi^- \rightarrow \mathrm{p}\bar{\mathrm{p}} \rightarrow \mathrm{pe}^-,$$

where the primordial particle (D) is described by Dirac's "new equation" (see Sec. 6.4) and $\tau_0^\pm$ and $\mu_0^\pm$ are spinless tauons and muons, respectively. The end product of the particle evolution is the stable proton–electron pair (pe^-). After its appearance, particle interactions are without exception governed by today's well-known standard model (SM).

The key question is how the conversions of an electron pair into a pion pair and a pion pair into a proton pair are energetically possible. The answer to this question is that the mass-generating Higgs boson also brings mass (or rest energy) from virtual leptons appearing in the photon propagators to the quarks that build the pion and proton. When acting as a conveyor of mass, the Higgs appears in one of three very light mass states depending on which charged lepton it interacts with (tauon, muon, or electron). See Sec. 6.2.

Thus, the Higgs boson plays two different roles in the history of particle evolution. In its heavy version, the primary purpose of the Higgs is to generate the weak mass of the Z and W bosons via the Higgs mechanism. In contrast, in its "flyweight" version, the Higgs plays a dominant role in the "Higgs–neutrino mechanism" for mass exchange between charged leptons and quarks, while its generation of minor contributions to the masses of the leptons is a secondary effect.

In nuclear physics, the role played by the light Higgs particle is of immediate practical interest. Like the elementary spin-1 bosons (the massless photon and the massive Z^0 and $W^\pm$ particles), the spin-0 Higgs boson is a force-mediating gauge particle. When associated with charged leptons, the Higgs has a very low mass. Consequently, it should produce a force of sufficiently long range to affect the orbits of the up and down quarks that compose the nucleons. Therefore, the existence of "flyweight Higgs" particles might explain several puzzling properties of the nucleons (see Sec. 4).

The precise knowledge of two numerical constants derived in Sec. 6.5,

$$B = 0.666\ 001\ 731\ 498 \tag{2}$$

and

$$B_0 = 0.978\ 396\ 4019, \tag{3}$$

characterizing the structure of an electron and a (today nonexistent) "spinless muon," respectively, is essential for the simulation to produce a sensible result.

For the value of the muon–electron mass ratio before the appearance of the strong force and the Higgs, one obtains

$$(m_\mu/m_e)^{\mathrm{QED}} = 1/B\alpha + 1/(1 - 2B\alpha) = 206.769\ 04. \tag{4}$$

When the evolution of particle masses are simulated in detail, taking into account corrections due to the muon's ejection of four Higgs particles and capture of a neutrino, one obtains the theoretical value,

$$m_\mu/m_e = 206.768\ 283\ 185(78), \tag{5}$$

with its error mainly deriving from the uncertainty in the fine-structure constant α. This value is in good agreement with the experimental 2006 CODATA value of 206.768 2823(52).

2 Simulating the evolution of particles

The law of conservation of energy explains why the universe has to be matter–antimatter asymmetric, and why there are three particle generations. However, to understand the details of the evolution of particles and their masses, one must turn to the law of conservation of momentum (see Sec. 6.5).

2.1 The expanding universe

When applying the law of conservation of momentum to space, one is led to the conclusion that the universe is expanding because space is created inside particles and is flowing out from them. Also, one may conclude that the amount of space created per unit time by a particle is proportional to the particle's energy.

Therefore, consider a sphere with radius r and volume $V = \frac{4}{3}\pi r^3$ that grows at the same rate as space is created by a particle at its center. Because the space created is proportional to the particle's energy, the volume grows at a steady rate. In other words, $\mathrm{d}V/\mathrm{d}t$ is constant. With a suitable definition of the "particle radius," r_0, one may write $\mathrm{d}V/\mathrm{d}t = 4\pi c r_0^2$, or

$$\mathrm{d}r/\mathrm{d}t = c r_0^2/r^2, \tag{6}$$

valid for $r \gg r_0$.

Suppose next that V, instead of containing one particle, contains N particles, which means that

$$\mathrm{d}r/\mathrm{d}t = cNr_0^2/r^2. \tag{7}$$

Particles on the "horizon" of the universe recede with velocity $\mathrm{d}r/\mathrm{d}t = c$, and the distance to the horizon is given by the universe's radius, defined as $R = c/H$, where H is the Hubble expansion rate. Letting V be the volume of the entire universe (i.e., setting $\mathrm{d}r/\mathrm{d}t = c$ and $r = R$), the number of particles in the universe is given by

$$N = R^2/r_0^2. \tag{8}$$

From Eq. (7), one obtains via $\int_0^r r^2 \mathrm{d}r = cNr_0^2 \int_0^t \mathrm{d}t$ the relation

$$r^3 = 3cNr_0^2 t, \tag{9}$$

where t is the age of the universe. Division of Eq. (7) by Eq. (9) gives $\mathrm{d}r/\mathrm{d}t = r/3t$ and, choosing $r = R$,

$$R = 3ct, \tag{10}$$

which implies that the radius of the universe grows linearly with time.

Thus, a volume containing a fixed number of particles ($V \propto r^3$) grows like t, whereas the volume of the universe ($V_u \propto R^3$) grows like t^3.

From Eqs. (8) and (10), it follows that

$$N \propto t^2, \tag{11}$$

where N is the number of particles in the universe and t is the age of the universe.

2.2 About the simulation program

The simulation relies on two primary assumptions, which follow from the application of the momentum equation to space, but which could as well be inferred from Dirac's large-number hypothesis (LNH) [1, 2, 3] deduced by Paul Dirac from observations that still hold true.

The first assumption states that, for a volume V coexpanding with the universe,

$$\mathrm{d}V/\mathrm{d}t = \text{const.}, \tag{12}$$

implying an unperturbable expansion of space.

The second assumption is that the expanding volume's energy content

$$E_V = \text{const.}, \tag{13}$$

implying that energy is globally conserved.

In addition to the two primary assumptions, a few secondary assumptions are needed. However, thanks to the information provided by the experimentally known values of the tauon–muon mass ratio (16.82), muon–electron mass ratio (206.768), and photon–baryon number ratio (a few billion), resorting to guesswork is not needed because mathematical experimentation unambiguously reveals what assumptions have to be made in order to obtain a both consistent and realistic model of the early universe. It turns out that, even though a couple of crude approximations are made, the simulation yields detailed information about the evolution of particles.

Assume for simplicity that the universe is created as a single, outward neutral pair of "spinless tauons" ($\tau_0^+\tau_0^-$), which pops up in a finite (i.e., nonzero) spacetime volume in a phase transition that breaks the perfect symmetry of literally nothing.

Thus, in phase 1, at the "time of creation" t_c, the universe contains $N_c = 1$ particle pairs occupying a volume V_c.

As space expands, the number of particles in the universe grows according to Eq. (11), at the same time as the massive particle pairs annihilate into pairs of massless photons. When the last spinless-tauon pair is about to annihilate (and thereby trigger the rematerialization of radiation in a second phase transition), the volume of the universe is V_1 and it contains N_1 particle pairs. Quantum indistinguishability means that each one of them may be regarded as originating from the very first particle pair.

In phase 2, history repeats itself, now with the pair of spinless muons ($\mu_0^+\mu_0^-$) playing the leading role. By the end of phase 2, the volume of the universe has grown from V_1 to V_2, and the number of particle pairs from N_1 to N_2.

In phase 3, history repeats itself once more, but now with the pair of spinning electrons ($\mathrm{e}^+\mathrm{e}^-$) in the leading role. Initially, phase 3 contains $2N_2$ electron pairs, since, in the third phase transition, every photon has rematerialized as two electrons (an electron–positron pair). At the end of phase 3, when the last pair of electron pairs are about to annihilate, the universe's volume has grown to V_3, and it contains $N_3 - 1$ photon pairs in addition to the remaining two electron pairs.

For each phase, assume a value N for the final number of particle pairs and calculate the time it takes for all N pairs to annihilate. Addition of this time interval to the time when the phase begins gives the time t (i.e., the age of the universe) when the phase ends. At this point in time, $N = (t/t_c)^2$ should hold true as required by Eq. (11). If necessary, repeat the calculation using another value of N and continue until the requirement (11) is met as closely as possible.

If the model is good, the self-energy of the massive particles should grow by a factor that in phase 1 matches the tauon–muon mass ratio, in phase 2 matches the muon–electron mass ratio, and in phase 3 is of order 10. Because of the sensitivity of the results to variations in input, it can be seen that, to meet these requirements, the following three assumptions must be made.

The lifetime of the massive particle pair and the speed of light increase at the same rate, that is, $\tau \propto c$.

At the beginning of phases 1 and 2, the particle lifetime equals the initial age of the universe, that is, $\tau = t_c$. This result follows from the simulation's requirement that, when expressed in units of t_c, the lifetimes of the two phases should acquire simple numerical values when the masses grow with factors matching those calculated from the measured mass ratios. One may also say that the result follows from the principle of maximum simplicity [4] because it is difficult to advocate any other initial value than 1 for τ/t_c. Alternatively, viewing the very first particle, not as a pair of spinless tauons (i.e., charged bosons), but as an oscillator (see Sec. 6.4), one may reason that its mass or rest energy is generated in an "upward oscillation" of duration t_c, and, consequently, will attempt to disappear in a subsequent "downward oscillation" an equally long instant t_c later.

Finally, for the simulation program to produce the correct mass ratios, one must assume that the number of particles (N) in the universe does not increase like $N = (t/t_c)^2$ as suggested by Eq. (11), but like

$$N = (t/t_c - 1)^2, \quad t \geq 2t_c. \tag{14}$$

This result can be interpreted to mean that the primordial particle is not a pair of spinless tauons as first assumed, but a massive neutral particle (D), which during its lifetime t_c remains alone in the universe (Sec. 6.4).

2.3 The spinless tauon reigns

Use the index c to refer to the first phase transition in which matter is initially created. Thus, the initial time (t_c), particle lifetime ($\tau_c = t_c$), speed of light (c_c), and energy (E_c) are the fundamental constants of creation. Further, use i to refer to a quantity's initial value in a phase, and f to refer to its final value, when the next phase transition is triggered.

At the beginning of phase 1, the values of $t_i = t_c, \tau_i = \tau_c$, and $c_i = c_c$ are given, while at the end of the phase, the values of t_f, τ_f, and c_f should result from the simulation.

Let N_f be the total number of particle pairs at the end of the phase. Further, introduce the time-dependent variables

$E_{m1} = mc^2$ representing the energy of a massive pair,
$N =$ the actual number of massive pairs,
$E_m = NE_{m1} =$ the total energy of massive pairs, and
$E_r =$ the total energy of the photon pairs.

It can be seen that, for the simulation to work properly, the mass m (in $E_{m1} = mc^2$) must be constant.

Without loss of generality, one may set m as well as the initial values of t, τ, and c (and thereby E_{m1}) equal to 1.

In phase 1, the requirement of the law of conservation of energy differs from Eq. (1) because quantized spin and the Planck constant h do not exist. Therefore, without the restriction imposed by the constancy of h in Eq. (1), $E_r \propto 1/\lambda \propto r_i/r$ should hold for the development over time of the radiative energy. Now, in a quantum universe, the radius r is an unmeasurable quantity whose use should be avoided in the simulation program. Replacing, therefore, r_i/r by $(t_i/t)^{1/3}$ according to Eq. (9), one may summarize the assumptions used in the simulation of phase 1:

$$\begin{aligned} &E_m + E_r = N_f, \\ &E_r \propto t^{-1/3}, \\ &\tau_i = t_c = 1, \\ &\tau = \tau_i c, \\ &N_f = (t_f/t_c - 1)^2. \end{aligned} \tag{15}$$

The logic of the program involves the following steps:

1. Assume a tentative value for N_f.
   Set E_m and N equal to N_f, $E_{m1} = 1$, and $E_r = 0$.
   Set t_c and $\tau_i = 1$.
   Set $t = t_c$ (equal to 1).
2. Obtain the lifetime τ from $\tau_i\sqrt{E_{m1}}$ (which equals $\tau_i c = c$).
3. Add to the age t a time increment of $\Delta t = \tau/N$ to obtain the age of the universe when the next particle pair annihilates.
4. Calculate the decrease in radiation energy during Δt: $\Delta E_r = E_r[1 - ((t - \Delta t)/t)^{1/3}]$.
5. Subtract ΔE_r from E_r and add it to E_m.
   Get the rest energy of one massive particle pair through $E_{m1} = E_m/N$.
6. If $N = 1$, go to step 8.
7. Decrement by 1 the number of massive particles N.
   Subtract E_{m1} from E_m and add it to E_r.
   Go back to step 2.
8. With $t_f = t$, calculate $t_f - \sqrt{N_f}$, which should approximately equal 1 at the same time as E_m as closely as possible should match its target value of 16.919 obtained from the tauon–muon mass ratio's present value of about 16.818 (see Sec. 3).

Since E_r at the end of phase 1 (when $E_m = 0$) has the same value as E_m at the beginning of the phase (when $E_r = 0$), the photon pairs have the same energy, E_c, as the massive tauon pair (or the D particle) had originally. Since this energy is conserved when the photon pairs "freeze," c returns to its original value of c_c, and the initial energy of the muon pair becomes equal to the energy E_c of the D particle.

The method used here cannot produce very precise results because of two approximations made in the simulation.

The first one is the invalid assumption that N_f is an integer when the phase ends at time t_f. In an improved calculation, one should note that the evolution of particles simultaneously proceeds along all possible paths toward stable proton–electron matter.

The second approximation is made in step 3, where Δt is obtained by using a loosely specified average value for the time-dependent lifetime τ. One way of obtaining such an average is to write $t_{av} = t + x\Delta t$, where x has a value near but less than 0.5 (such as $x = 0.43855$), initialize Δt to $\Delta t = \tau/N$, and repeat the following steps ten times:

$$
\begin{aligned}
t_{av} &= t + x\Delta t, \\
\Delta E_r &= E_r(1 - (t/t_{av})^{1/3}), \\
E_{m1} &= (E_m + \Delta E_r)/N, \\
\tau &= \tau_i\sqrt{E_{m1}}, \\
\Delta t &= \tau/N.
\end{aligned}
\tag{16}
$$

The correct method should be to constantly relate the time t (or the universe's age) of the global picture to the time (or age) t' of the local, standard picture, in which particle lifetimes are constant and, consequently, $\Delta t' = \tau'/N$ with $\tau' = 1$.

When combined with the simulation of phase 2, the simulation of phase 1 — the spinless-tauon phase — yields for the end of the phase the following approximate values for the number of particle pairs, the age of the universe, and the factor by which the massive particles have increased in rest energy:

$$\begin{aligned} N_1 &\approx 86, \\ t_1 &\approx 10, \\ f_1 &= c^2 \approx 16.9. \end{aligned} \tag{17}$$

2.4 The spinless muon takes over

Phase 2 begins with the rematerialization of the N_1 photon pairs that are the end product of phase 1. Conservation of energy demands that the pairs of massive "spinless muons" acquire the same rest energy, E_c as the corresponding pairs of phase 1 had initially.

The spinless tauons continue to exist as virtual particles, and the tauon–muon mass ratio has a value (f_1) near the target value of 16.919 used in simulating phase 1.

For phase 2, the same assumptions hold that are made for phase 1 in Eq. (15). Also, the program logic is the same. However, the initial time, $t = t_i$, is now equal to $t_1 = t_f$ at the end of phase 1, and the value of N_f is again found via trial and error.

In addition, the target value 16.919 mentioned in step 8, is replaced by the value 151.136 obtained from the uncorrected theoretical value $1/B\alpha = 205.759$ of the muon–electron mass ratio (see Sec. 3).

The computation shows that $N_f = (t_f - 1)^2$ — but not $N_f = t_f^2$ — can be simultaneously satisfied in both phases, and that this happens when N_f is adjusted to make the calculated increase in self-energy of the particles approximately match the theoretical target values (16.919 in phase 1 and 151.136 in phase 2).

For the end of phase 2 (the spinless-muon phase), the simulation yields approximate values for the number of particle pairs, the age of the universe, and the factor by which the massive particles have increased in rest energy:

$$\begin{aligned} N_2 &\approx 1000, \\ t_2 &\approx 33, \\ f_2 &= c^2 \approx 151.1. \end{aligned} \tag{18}$$

Hopefully, a stringent approach to the simulation of the first two phases will produce exact values for the increase in particle rest energy, and thereby lead to precisely predicted values for the today only experimentally known mass ratios m_τ/m_μ and m_μ/m_e — and from the latter ratio, a theoretical value for the fine-structure constant α.

2.5 The electron appears

In phase 3, N is the number of pairs of electron pairs (since in the third symmetry-breaking transition $\gamma_\mu\gamma_\mu \rightarrow \mathrm{e^+e^-\ e^+e^-}$, whereas in the second transition $\gamma_\tau\gamma_\tau \rightarrow \mu_0^+\mu_0^-$).

The overall logic of the simulation is the same as that of phases 1 and 2. However, the electromagnetic force now replaces the electric force of the earlier phases. Therefore, since the Planck constant h has appeared in company with the quantized spin of the electron, Eq. (1)

holds true and the assumptions become

$$\begin{aligned} &Nc^2 + pc(t_i/t)^{1/3} = N_f, \\ &E_r = pc(t_i/t)^{1/3}, \\ &\tau_i = \frac{1}{8\pi}\alpha^{-2} = 747.1873, \\ &\tau = \tau_i c, \\ &N_f = (t_f/t_c - 1)^2. \end{aligned} \tag{19}$$

The logic of the program involves the following steps:

1. Assume a tentative value for N_f.
   Set E_m and N equal to N_f, $E_{m1} = 1$, and E_r and $p = 0$.
   Set $t_c = 1$ and $\tau_i = \frac{1}{8\pi}\alpha^{-2} = 747.1873$.
   Let t retain its value (equal to t_f of phase 2).
2. Obtain the current value of c from the first equation (which is of second order in c).
   Get $\tau = \tau_i c$.
3. Add to the age t a time increment of $\Delta t = \tau/N$ to obtain the age of the universe when the next particle pair annihilates.
4. If $N = 1$, go to step 8.
5. Decrement by 1 the number of massive particles N.
   Get $E_m = Nc^2$ and $E_r = N_f - E_m$.
6. Obtain a new value for p from the second equation (i.e., $p = E_r c^{-1}(t_i/t)^{-1/3}$).
7. Go back to step 2.
8. With $t_f = t$, calculate $t_f - \sqrt{N_f}$, which should equal 1.
   If it does not, try another value for N_f and repeat the above steps.

Adjusting N_f of each phase in such a way that the target values for phase 1 and phase 2 are approximately met and $t_f - \sqrt{N_f}$ as closely as possible equals 1 (the only choice that leads to consistent results) for all three phases, simulation of phase 3 — the electron phase — yields for the end of the phase the following approximate values for the number of particle pairs, the age of the universe, and the factor by which the massive particles have increased in rest energy:

$$\begin{aligned} &N_3 = 1393\ 137\ 450, \\ &t_3 = 37\ 326, \\ &f_3 = c^2 = 10.535. \end{aligned} \tag{20}$$

For the initial value of the photon–baryon number, one finds (with $n_b = 1$ proton present immediately after the final transition to stable proton–electron matter has taken place) the approximate value

$$n_\gamma/n_b = 2(N_3 - 1) = 2786\ 274\ 900. \tag{21}$$

These results are obtained without use of arbitrarily adjustable parameters. That is, the particle lifetimes have the natural value of 1 in terms of the basic time unit t_c with the exception of the deduced lifetime,

$$\tau_3 = \frac{1}{8\pi}\alpha^{-2} = 747.1873, \tag{22}$$

of the four-electron particle state of phase 3. The expression for τ_3 given in Eq. (22) is a prediction of the model, which I have not tried to verify via direct calculation. However, the fact that the factor $\frac{1}{\pi}\alpha^{-2}$ appears in calculations of electron–positron annihilation [5, 6] makes the expression plausible.

Hopefully, a parallel calculation of the lifetimes of the entangled pair of electron–positron pairs in phase 3 and the entangled pair of scalar bosons appearing in the earlier phases will confirm the ratio, $1/8\pi\alpha^2$ to 1, between the two values.

Also, theoretical considerations should relate the predicted lifetime, $747.1873\,t_c$, to measured lifetimes. For instance, the lifetime of parapositronium is obtained in Eq. (5–128) on p. 234 in Ref. [6] as $\frac{2}{m}\alpha^{-5} = 1.25\times10^{-10}$ s, while Eq. (5–125) on p. 233 shows that the expression for the annihilation lifetime contains the factor $\frac{1}{\pi}\alpha^{-2}$. Naively assuming that the two lifetimes are related via $\alpha^{-2}/\alpha^{-5} = 1/137.036^3 = 0.389 \times 10^{-6}$, one arrives at an annihilation lifetime of 0.486×10^{-16} s and a value of 0.650×10^{-19} s for t_c. Consequently, a tentative assumption might be that t_c is of magnitude

$$t_c = 10^{-19} \text{ s.} \tag{23}$$

In summary, without resorting to freely adjustable parameters, a simulation based on the two primary assumptions in Eqs. (12) and (13) can in a consistent and plausible way explain the existence of the tauon, muon, and electron as well as their mass ratios (m_τ/m_μ and $m\mu/m_e$) and the photon–baryon number ratio (n_γ/n_b).

2.6 Via pion pairs to proton pair

The logical next step in the evolution of particles should be that a proton–antiproton pair ($\mathrm{p\bar{p}}$) replaces the positron–electron pair ($\mathrm{e^+e^-}$). However, the simulation of the particle evolution only makes sense if the transition involves an intermediate "pion parenthesis" and proceeds in two steps; first $\mathrm{e^+e^-}\ \mathrm{e^+e^-} \to \pi^+\pi^-\ \pi^+\pi^-$ and, after one of the pion pairs has annihilated, $\pi^+\pi^- \to \mathrm{p\bar{p}}$.

For this process to be energetically possible, the energy

$$E_1 = 4(m_\pi - m_e)c^2 = 556.236 \text{ MeV} \tag{24}$$

and, an instant later,

$$E_2 = 2(m_p - m_\pi)c^2 = 1597.404 \text{ MeV}, \tag{25}$$

must be obtained from the background photons.

In the transition $\mathrm{e^+e^-}\ \mathrm{e^+e^-} \to \pi^+\pi^-\ \pi^+\pi^-$, the Higgs boson enters the scene for the first time. The lepton mass, which in phase 3 in its entirety is generated by the photon field, is split into two components. The main component is still generated by the electromagnetic photon field, but the second mass component is generated by the weak Higgs field and equals the mass of the Higgs boson itself (see Sec. 6.2).

In a first approximation, one may ignore the very light Higgs particles associated with the muon and electron, and only consider the tauon-type Higgs of mass $0.505\ \mathrm{MeV}/c^2$.

The probability for the photon to form a tauon pair in its propagator is $\alpha \approx 1/137$, where α is the fine-structure constant.

The purpose of the first Higgs particle is to enable the building of physical pion pairs by conveying the required energy, $E_1 = 556.236$ MeV, from the virtual leptons appearing in the propagators of the $n_\gamma = 2786$ million background photons to the u and d quarks that build the pions ($\pi^+ = \mathrm{u\bar{d}}$, $\pi^- = \mathrm{d\bar{u}}$).

The conserved total energy of all particle pairs is the same at the beginning and end of phase 3. However, at the end of the phase, the self-energy $4m_ec^2$ of the two remaining electron pairs have increased by a factor of $f_3 = 10.535$. This increase in energy of the two electron pairs is counterbalanced by the same decrease in energy distributed among the final $N_3 - 1$ $= 1393$ million photon pairs, which means that the relative decrease in photon energy during phase 3 is negligibly small (and would have disappeared altogether if the last two electron pairs had annihilated, too).

In the local picture, where $m_e c^2 =$ constant, things look different. Thus, at the end of phase 3, there are $2(N_3 - 1) = 2786\ 275\ 000$ background photons, each possessing an energy of $2m_e c^2/f_3 = 2 \times 0.511$ MeV$/10.535 = 97.010$ keV, which means that they have lost over 90 percent of their initial energy.

When the tauon ejects a Higgs particle, the relative decrease in tauon mass is $m_{H_\tau}/m_\tau = G_F m_\tau^2/4\sqrt{2}\pi\alpha$ (see Sec. 6.2).

Since the probability for the photon to be in the tauon state (that is, forming a tauon loop in its propagator) is $\alpha \approx 1/137$, the relative decrease in energy of the background photons is $G_F m_\tau^2/4\sqrt{2}\pi = 2.072\,46 \times 10^{-6}$. That is, the ejection of a set of Higgs particles by the photons causes a total decrease of photon energy that amounts to $2(N_3 - 1) \times (2m_e c^2/f_3) \times G_F m_\tau^2/4\sqrt{2}\pi$ $= 560.2$ MeV.

This value, which is obtained through simulation in the global picture, should be compared to the local picture's value of $E_1 = 4(m_\pi - m_e)c^2 = 556.236$ MeV.

The discrepancy between the two energies implies that the energy-balance equation,

$$2(N_3 - 1)(2/f_3)m_e G_F m_\tau^2/4\sqrt{2}\pi = 4(m_\pi - m_e), \tag{26}$$

is not satisfied at the time of creation of the pion. However, this result does not prove that the simulation is flawed.

Instead, noting that a small additional growth of f_3 would make the equation hold true, the result may be interpreted to mean that the comparison between energies is made too early, and that the simulation must continue until the real proton has gotten its mass. Only then it is possible to compare energies of the particles appearing in the simulation with the energies of presently existing particles.

Actually, the continued simulation shows that, after about 1000 t_c, or according to Eq. (23), about 10^{-16} s, the rest energy of the last pion pair has grown sufficiently large for Eq. (26) to be satisfied.

The conclusion can only be that the last pion pair did not decay via the fast strong interaction (in something like 10^{-24} s), but via the much slower weak interaction. This discovery, in turn, reveals the purpose of each weakly interacting particle.

2.7 The Higgs–neutrino mechanism

The overall picture of the evolution of matter suggested by the computer simulation may be summarized as

D	$\to$	$\tau_0^+\tau_0^-$	$\to$	$\underline{\mu}_0^+\underline{\mu}_0^-$	$\to$	$\mu_0^+\mu_0^-$	$\to$	$\underline{e}^+\underline{e}^-$	$\to$	e^+e^-	$\to$	Higgs–neutrino mechanism	$\to$	$p\bar{p}$	$\to$	pe^-
1		9		0		23		0		37 293		1000		0		∞
1		10^2		10^2		10^3		2×10^3		3×10^9						

The underlined symbols indicate newborn, "frozen" particles that immediately turn into dynamically interacting particles. The first row of numbers indicates time elapsed during each stage of the evolution of matter in units of $t_c \approx 10^{-19}$ s. The second row of numbers displays the quantity of particles in each phase; hence, phase 1 begins with one D particle and ends with (about) 100 photon pairs resulting from tauon-pair annihilation; phase 2 begins with 100 spinless-muon pairs and ends with (about) 1000 photon pairs; and phase 3 begins with 2000 electron pairs (rematerialized pairwise from pairs of photons, $\gamma_\mu\gamma_\mu \rightarrow \mathrm{e^+e^-\ e^+e^-}$) and ends with about 3 billion background photons.

When interpreted as being caused by a prolonged duration of the pion state, the discrepancy between the two sides of Eq. (26) suggests that the Higgs–neutrino mechanism works as follows:

$$\mathrm{e^+e^-\ e^+e^-} \rightarrow \underset{0}{\underline{\pi}^+\underline{\pi}^-\ \underline{\pi}^+\underline{\pi}^-} \xrightarrow{\mathrm{H}} \underset{10^{-5}}{\pi_-^+\pi_-^-\ \pi_-^+\pi_-^-} \rightarrow \underset{10^{-5}}{\pi_-^+\pi_-^-} \xrightarrow{\mathrm{Z}} \underset{10^{3}}{\pi_+^+\pi_-^-} \rightarrow \underset{10^{-5}}{\pi_+^+\pi_+^-} \xrightarrow{3\mathrm{H}\nu\mathrm{W}} \underset{0}{\mathrm{p_+\bar{p}_+}} \rightarrow \underset{\infty}{\mathrm{p_+e_+^-}}$$

The intrinsic parity associated with the particles are indicated in subscripts. Again, the numbers below the particle states indicate time duration in terms of the basic time unit t_c, suggesting that the transformation of unstable electronic matter ($\mathrm{e^+e^-}$) to stable proton–electron matter ($\mathrm{pe^-}$) takes about $1000\,t_c$ or 0.1 fs (10^{-16} seconds).

The purpose of the first Higgs boson is to enable the up and down quarks to form four real (that is, mass-bearing physical) pions by delivering to them the energy $4(m_\pi - m_e)c^2$, which is needed to make the transition $\mathrm{e^+e^-\ e^+e^-} \rightarrow \pi^+\pi^-\ \pi^+\pi^-$ possible.

Within less than $10^{-5}\,t_c$ (or about 10^{-24} s), one of the pion pairs annihilates via strong interaction.

After a lapse of another $10^{-5}\,t_c$, the imminent annihilation of the remaining pion pair is prevented by the neutral Z boson, which comes to the rescue and switches the parity of one of the pions ($\pi_-^+ \rightarrow \pi_+^+$), thereby inhibiting strong decay of the pair.

Since the appearance of the Z particle does not affect the lepton and quark masses (see Sec. 6.6), the event obeys the law of conservation of energy.

About $10^3\,t_c$ later, the weak parity-switching force brought by the Z boson causes a second, now spontaneous change of pion parity, which again enables strong decay of the pion pair ($\pi^+\pi^- \rightarrow \gamma\gamma$).

With the universe's pionic matter doomed to extinction, it has to be replaced by protonic matter, which requires a second energy transfer that is $2(m_p - m_\pi)c^2/4(m_\pi - m_e)c^2 = 2.872$ times larger than the first transfer effectuated by one set of Higgs particles. Therefore, three sets of Higgs particles are assigned the task of delivering additional energy to the quarks.

Since part of this energy remains unused by the quarks, it has to be restored to the virtual leptons by neutrinos (ν), which to be able to accomplish their task call for help from charged W bosons. Also, to hand over their mass, which is initially given as 0.128 times the Higgs mass (that is, about 64.6 keV/c^2, 13.6 eV/c^2, and 1.54 μeV/c^2), in the proportions demanded by the receiving leptons, the neutrinos must be able to change their mass in flight. Since their primary mission — to relieve the quarks of unused mass — is fulfilled at the instant the neutrinos are ejected by the quarks, the time between their ejection and capture (their time of flight) required for the mass oscillation to take place is irrelevant.

Naively, one might imagine the Higgs as a photon transformed into a standing wave forming a closed string with its energy or mass determined by the frequency of the wave. The neutrino, in turn, would be formed from three traveling waves, which gives it a continuously changing mass and a spin of 1/2. The principle of maximum simplicity suggests that the neutrino is its own antiparticle.

2.8 The superweak force

The comparatively long time that elapses between the two parity-switching events ($\pi^+_-\pi^-_- \to \pi^+_+\pi^-_-$ and $\pi^+_+\pi^-_- \to \pi^+_+\pi^-_+$) introduces an asymmetry between pion ($\pi^+ = \mathrm{u\bar{d}}$) and antipion ($\pi^- = \mathrm{\bar{u}d}$) — that is, between matter and antimatter. Presumably, this asymmetry implies that the down and anti-down quarks are not exact opposites of each other, which might explain the mysterious so-called "superweak force" observed in the decay of neutral kaons and B mesons (both of which contain a down quark: $\mathrm{K}^0 = \mathrm{d\bar{s}}$, and $\mathrm{B}^0 = \mathrm{d\bar{b}}$).

2.9 The antiproton big bang

Being fermions, the proton and antiproton have opposite parity — positive and negative, respectively. However, the conclusion in Sec. 2.7 is that both component particles of the first real proton pair inherit positive parity from the last real pions. Consequently, the particle pair is not physically viable and would annihilate immediately if the result — a massless universe — was allowed. Since annihilation is forbidden by the law of conservation of energy, the universe undergoes a phase transition obeying the principle of least symmetry breaking. That is, with its charge, spin, and positive parity conserved, the negatively charged component of the particle pair is forced to transform into an electron.

Every enforced transition introduces a new element or feature that adds to the complexity of the world. This time, the law of conservation of energy causes introduction of motion and thereby kinetic energy, which heats the now stable matter (pe^-) to about a trillion Kelvin (10^{12} K) in a nuclear "big bang." As a consequence, the second law of thermodynamics enters the scene.

Since the transition is a unique, non-repeatable leap from a forbidden, unphysical state to the nearest allowed physical state, no channel for (anti)proton decay is created in the process. Therefore, it follows from the principle of maximum simplicity that the proton is stable.

2.10 Fine-tuning the model

This far, only contributions to the pion and proton masses originating from tauon pairs appearing in the photon propagators have been considered. Taking into account the contributions coming from muon pairs gives a small correction to the result of the simulation.

Considering the theoretical explanation for the muon–electron mass ratio (see Sec. 3), and using its experimental 2006 CODATA value of

$$(m_\mu/m_e)^{\mathrm{exp}} = 206.768\ 2823(52), \tag{27}$$

the details of the evolution from electron pairs to the final proton pair become precisely determined [7].

Amazingly, the simulation returns a value for m_μ/m_e that agrees with, and is nearly two orders of magnitude more accurate than the CODATA value, namely

$$m_\mu/m_e = 206.768\ 283\ 185(77)(7)(5)(5), \tag{28}$$

where the uncertainties come from errors in $\alpha^{-1} = 137.035\ 999\ 084(51)$, G_F, m_π, and m_τ, respectively.

3 The muon–electron mass ratio

By applying the momentum equation to space, one is led to assume that to lowest order the muon–electron mass ratio has the value $1/B\alpha = 205.759\ 22$, which is obtained in Eq. (77) in Sec. 6.5. The difference of 1.009 06 between this value and the measured 206.768 28 lies near $1 + 2B\alpha = 1.009\ 72$, suggesting that $m_\mu/m_e = (1/B\alpha)[1 + B\alpha + 2B\alpha + \ldots]$.

In Appendix A.3 this relation is explained. There, studying in detail the process in which the spinless muon of phase 2 acquires spin in the third phase transition, one obtains for the theoretical mass ratio the value

$$m_\mu/m_e = 1/B\alpha + 1/(1 - 2B\alpha) = 206.769\ 04, \tag{29}$$

which implies that the muon of today has lost a small part of its original mass ($206.769\ 04 - 206.768\ 28 = 0.000\ 76$ in units of m_e).

The value in Eq. (29) includes the Higgs contribution to the mass ratio, which is

$$(\Delta m_\mu/m_e)^{\mathrm{H}} = 0.000\,2076, \tag{30}$$

and which also gives the amount of mass carried away by a single Higgs particle ejected by the muon. See Sec. 6.2. Thus, the ejection of four Higgs particles would cause a correction of $-0.000\ 83$ to m_μ/m_e, leaving a positive contribution of about 0.000 07, or

$$(\Delta m_\mu/m_e)^{\nu} \approx 0.34\,(\Delta m_\mu/m_e)^{\mathrm{H}}, \tag{31}$$

deriving from the mass brought back by the neutrino.

The observation that $1 + 2(m_p - m_\pi)/4(m_\pi - m_e) = 3.872 = 1 + 3 - 0.128$ (see Sec. 2.6) suggests that the transition from electron pairs to a proton pair is performed in two steps, $\mathrm{e^+e^-\ e^+e^-} \rightarrow \pi^+\pi^-\ \pi^+\pi^-$ and $\pi^+\pi^- \rightarrow \mathrm{p\bar{p}}$, with one plus three sets of Higgs particles bringing mass to the quarks, and one set of neutrinos returning unused mass.

The simulation demonstrates that this assumption is consistent with the tauon-type Higgs possessing a mass of $m_{H_\tau} = 0.505$ MeV/c^2 (see Eq. (44) in Sec. 6.2), and the tauon-type neutrino returning a mass of $0.128\,m_{H_\tau} = 64.6$ keV/c^2 to the tauon loops appearing in the propagators of the background photons.

Now, the relative, unused portion of the Higgs mass that the quarks give over to the neutrino should be independent of the actual mass state of the Higgs. That is, the initial neutrino mass should be

$$(m_\nu)^{\mathrm{initial}} = 0.128\,m_H. \tag{32}$$

The fact that the mass received by the muon is $0.34\,m_{H_\mu}$ and not $0.128\,m_{H_\mu}$ implies that the neutrino, in its interaction with the charged leptons, appears with mass ratios different from those of the Higgs, for which holds that

$$m_{H_e} : m_{H_\mu} : m_{H_\tau} = m_e^3 : m_\mu^3 : m_\tau^3. \tag{33}$$

Noting that $0.34/0.128 = 2.66$ approximately equals $\log(m_\tau/m_\mu) = \log 16.82 = 2.82$ suggests that, when the muon-type neutrino is captured by the muon, it does not deliver its initial mass of $(m_\mu/m_\tau)^3\, m_{\nu_\tau}$ to the muon, but a mass amounting to

$$(\Delta m_\mu)^{\nu} = (m_{\nu_\mu})^{\mathrm{capture}} = (m_\mu/m_\tau)^3 \log(m_\tau/m_\mu)\, m_{\nu_\tau}. \tag{34}$$

When the simulation is repeated, taking into account the mass contributed by the muon in building the pion and proton pairs, and Eq. (34) is used, it turns out that [7]

$$m_\mu/m_e = 206.768\ 283\ 185(78) \tag{35}$$

must hold for the adjusted version of the balance equation (26) to be satisfied. The perfect agreement of this very accurate value with the measured value of 206.768 2823(52) suggests that the relation in Eq. (34) is not accidental. If that is so, it might hint at which model one should select among the many weak models that are being proposed.

The target values for the tauon–muon and muon–electron mass ratios differ from their present values. Thus, the value

$$(m_\tau/m_\mu)^{\text{target}} = 16.919 \tag{36}$$

used in the simulation of phase 1 is larger than today's value of $m_\tau/m_\mu = 16.82$, which reflects the fact that the tauon has lost part of its original mass.

The target value for the muon–electron mass ratio,

$$(m_\mu/m_e)^{\text{target}} = B_0/2B^2\alpha = 151.136, \tag{37}$$

follows from division of the ratio's initial value of $1/B\alpha$ by the factor $2B/B_0$, which says that a single spin-0 muon in phase 2 corresponds to a pair of electrons in phase 3 and is B_0/B times heavier than the spin-$\frac{1}{2}$ muon of phase 3.

4 Nuclear physics: Higgs explains puzzling observations

A comparison between the Higgs and photon Feynman diagrams leads one to conclude that the Higgs-mediated force between two charged particles is repulsive, independent of the signs of the charges (whereas the photon-mediated electromagnetic force is attractive or repulsive depending on whether the charges are of opposite or equal sign).

The fact that no repulsive static force attributable to the Higgs has been observed suggests that the Higgs-mediated force is of purely dynamic nature, possibly resembling the photon-mediated magnetic force.

In nuclear physics, there are a number of puzzling observations which might be explained by the existence of a very light Higgs particle that produces a force of sufficiently long range to affect the behavior of the constituent particles of the nucleons.

Thus, theoretically predicted values for the magnetic moments of the proton, neutron, and other hadrons tend to be considerably smaller than the experimentally observed values [8].

A simple explanation for this discrepancy might be that, when acting on the quarks in a hadron, the repulsive Higgs force causes the quarks to move in as wide orbits as allowed by the strong force (which, like the force of a rubber band, increases with growing distance between the quarks).

The same explanation should apply to the so-called proton spin crisis, according to which the sum of the quarks' estimated orbital angular momenta and their measured spin angular momenta is less than the proton's spin.

A third problem in nuclear physics is that measurements of the proton radius using ordinary hydrogen and muonic hydrogen yield nonmatching results [9]. The straightforward explanation is that the light muon-type Higgs, whose existence the Brookhaven muon $g-2$ experiment convincingly demonstrates (see Sec. 6.3), causes a repulsive force between the quarks of the proton and the muon orbiting it.

5 Cosmological considerations

The computer simulation should be redone in such a way that the global time, in which particle lifetimes and duration of atomic-clock ticks increase, is continuously related to our standard local time, which we measure by counting clock ticks.

Also, the simulation should be continued into the present phase of the universe's development, and describe the evolution of structure after the proton and electron — the components of hydrogen — have taken over as mass-bearing particles.

With the appearance of kinetic energy and non-entangled protons and electrons, the gravitational force begins to influence the dynamical behavior of particles. Being initially perhaps 10^{30} times stronger than today, gravity will soon cause practically all of the universe's matter and radiation to form and be trapped by microscopic black holes, which later will either evaporate as G decreases or merge into successively more massive holes.

Via the action of gravity, small structures will combine to form larger structures. That is, black holes will form miniature galaxies, clusters of galaxies, etc. of ever-increasing size.

As long as the force of gravity is still very strong, structures spanning a considerable part of the entire universe (but not all of it, since gravity ultimately turns repulsive for distances approaching the radius of the universe) are held together by the gravitational force and do not participate in the universe's overall expansion.

Consequently, for a long time, the bulk of the existing radiation remains confined within the opaque structures, and is thereby prevented from losing energy through redshifting. Therefore, during this period, the length of the atomic-clock tick increases very slowly in the global picture.

Only after gravity has lost its grip on the big structures (today's galaxy clusters are already beginning to expand) and the universe has become transparent, the background radiation released from evaporated black holes will be able to travel freely between structures, thereby losing energy and causing the atomic-clock tick to lengthen at a comparatively rapid rate.

In other words, seen from the standard (or local) point of view, the universe spends a very long time (very many clock ticks) in its opaque phase. As a result, G (for which $\dot{G}/G = \dot{H}/H = -1/t = -3H$ would hold if no radiation redshift occurred and the global and local times coincided) decreases very slowly today.

As in the first phases, the simulation is in principle very simple, being nothing but a program loop around an elaborated version of Eq. (1) — noting, for instance, that the relativistic neutrino simultaneously loses kinetic energy and gains rest energy — in which one continuously monitors how energy subject to redshifting is transformed into rest energy and vice versa.

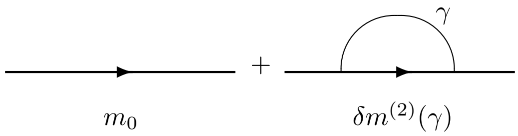
In sum, the simulation suggests that today's dark matter consists of originally microscopic black holes that merged with other black holes at a sufficiently fast rate to avoid evaporation as G decreased.

Also, it suggests that the recently (in the 1990s) discovered unexpected way in which the luminosity of type Ia supernovae appears to vary with distance is due to changes in supernova brightness caused by the decreasing G.

6 Details

6.1 The JBW theory

According to QED theory, the mass m of the electrically charged lepton (e, μ, or τ) is the sum of the particle's bare mass (m_0) and its self-mass (δm): $m = m_0 + \delta m$.



The graph pictures the bare mass of the charged lepton and its second-order photon correction.

However, in the standard theory, the value of m_0 is undetermined and δm is infinite. Considering only the contributions shown in the figure, one obtains for the renormalized mass m of the charged lepton

$$m = m_0 + \delta m^{(2)}(\gamma) + \ldots = m_0 + \left[\frac{3\alpha}{2\pi} \ln \frac{\Lambda}{m} + \ldots\right] m, \tag{38}$$

where Λ is a UV cutoff mass introduced to make the mathematics finite.

With m_0 unknown and the logarithm divergent, Eq. (38) appears to be void of physical content.

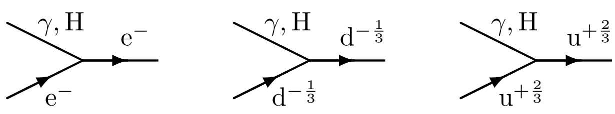
However, in the early 1960s, Kenneth Johnson, Marshal Baker, and Raymond Willey developed a perturbation theory (PT) "within the usual formalism of quantum electrodynamics" in which the electron's mass equals its self-mass ($m = \delta m$), implying that "the electron mass must be totally dynamical in origin" [10].

The JBW theory appears to have been quickly forgotten, and is rarely discussed in text books. For instance, Claude Itzykson and Jean-Bernard Zuber [6] devote three lines to it on page 424.

The most important implication of the JBW theory is that a pure QED universe is physically possible. In such a universe, which is inhabited by photons and electrically charged leptons (electrons, muons, and tauons), the photon field is the generator of all mass.

6.2 The Higgs and photon fields

The Feynman diagrams of SM [11] reveal that the Higgs particle (H) is a massive, weakly interacting spin-0 boson that accompanies and mimics the massless spin-1 photon (γ) in its interactions with the building blocks of ordinary matter (up and down quarks and electrons):



identical-looking diagrams picture photon and Higgs

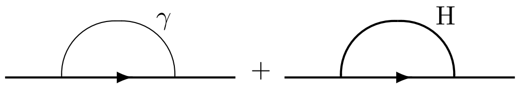
The similarity between the two particles suggests that the Higgs may be looked upon as a kind of spinless massive photon.

Even if the neutral spin-1 Z^0 particle couples graphically to quarks and charged leptons in the same way as the photon and Higgs do, it differs significantly from them because it also couples to the neutral ν (neutrino) lepton.

It follows from the JBW theory (see Sec. 6.1) that the photon field generates the electromagnetic mass components of the electrically charged elementary particles — the $W^{\pm}$ boson, the quarks, and the charged leptons (in the pure QED universe of phase 3, the entire mass of the charged lepton).

Since the spin-0 Higgs boson imitates the interactions of the spin-1 γ boson, a natural conclusion is that it generates weak mass in the same way as the photon generates electromagnetic mass.

Considering in parallel the second order mass contributions from the photon and Higgs to the charged lepton,



one obtains from standard electroweak theory the expression (see Appendix A.4)

$$m = m_0 + \delta m^{(2)}(\gamma) + \delta m^{(2)}(H) = m_0 + \frac{3\alpha}{2\pi} \ln \frac{\Lambda}{m} \left[1 + \frac{G_F m^2}{4\sqrt{2}\pi\alpha} \right] m, \tag{39}$$

for the renormalized mass m of the charged lepton.

Ignoring possible contributions from other particles and relying on the JBW hypothesis, which says that $m_0 = 0$, one immediately gets from Eq. (39) the ratio

$$\frac{\delta m^{(2)}(H)}{\delta m^{(2)}(\gamma)} = \frac{G_F m^2}{4\sqrt{2}\pi\alpha} \tag{40}$$

between the two contributions to the mass of the charged lepton.

In elementary-particle PT, the Higgs–lepton diagrams exactly parallel the photon–lepton diagrams. Therefore, one may expect that the ratio between the Higgs and photon contributions to the lepton mass should remain the same as in Eq. (40) also when higher-order self-mass diagrams are considered. In other words,

$$\frac{\delta m(H)}{\delta m(\gamma)} = \frac{G_F m^2}{4\sqrt{2}\pi\alpha} \tag{41}$$

is expected to hold true.

The Higgs and other possible corrections to the mass of the charged lepton are small in comparison with the photon's contribution to it. Therefore, one may set $\delta m(\gamma)$ equal to m in the left denominator of Eq. (41) and obtain as a good approximation

$$\delta m(H) = \frac{G_F m^2}{4\sqrt{2}\pi\alpha} m. \tag{42}$$

This result does not say anything definite about the mass of the Higgs particle itself. However, by simply assuming that the Higgs mass is directly calculable using standard methods, one may conclude — even without supporting evidence from the computer simulation — that the only value that can reasonably be advocated is $m_H = \delta m(H)$, or

$$m_H = \frac{G_F m^2}{4\sqrt{2}\pi\alpha} m, \tag{43}$$

where m represents the mass of m_τ, m_μ, or m_e, and m_H is the corresponding Higgs mass associated with each type of lepton.

For the mass of a Higgs particle emitted by a tauon, muon, and electron, respectively, one obtains from Eq. (43)

$$\begin{aligned} m_{H_\tau} &= 0.505 \text{ MeV}/c^2, \\ m_{H_\mu} &= 106.086 \text{ eV}/c^2, \\ m_{H_e} &= 12.0007 \ \mu\text{eV}/c^2. \end{aligned} \tag{44}$$

The self-energy $m_{H_e}c^2 = 12.0007\ \mu$eV of the electron-type Higgs corresponds to the energy of a photon of frequency 2.9018 GHz.

The weakly interacting “flyweight Higgs” particle is unstable, and annihilates into a pair of photons via, for example, a virtual charged-lepton loop. Also, at its first encounter with matter (electrons or quarks), it will most probably transform into a pair of ordinary photons. Therefore, its direct observation may not be possible.

Indirectly, however, it should be possible to detect it via its expected ability to penetrate much deeper into a medium than photons of comparable energy are able to do.

Also, the Higgs should cause a repulsive dynamic force resembling the magnetic force mediated by photons. As discussed in Sec. 4, this force might be revealing its presence in various puzzling observations in nuclear physics.

However, the hitherto most direct and therefore most convincing evidence for the existence of a very light Higgs boson comes from the Brookhaven muon $g-2$ experiment, which indicates that the Higgs appearing in the muon propagator is lighter than the muon itself ($m_{H_\mu} < m_\mu$).

6.3 The Brookhaven muon $g - 2$ experiment

The E821 muon ($g-2$) experiment at Brookhaven [12] yielded a very precise value for the muon's anomalous magnetic moment, a_μ (which is half of the muon's so-called $g-2$: $a_\mu = (g-2)/2$). The value that was obtained is

$$a_\mu^{\text{exp}} = 0.001\ 165\ 920\ 80(63). \tag{45}$$

If the particle (H_μ) appearing in the one-loop Higgs correction to a_μ is considerably lighter than the muon ($m_{H_\mu} \ll m_\mu$), it contributes theoretically with (see Appendix A.5)

$$a_\mu(\text{H}_\mu) = \frac{3G_F m_\mu^2}{8\sqrt{2}\pi^2} = 0.000\ 000\ 003\ 50 \tag{46}$$

to the value of a_μ. Together with the rest of the contributions, it gives

$$a_\mu^{\text{th}} = 0.001\ 165\ 921\ 28(61) \tag{47}$$

for the theoretical a_μ value. The difference

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = -0.000\ 000\ 000\ 48(88) \tag{48}$$

indicates good agreement between the theoretically predicted and experimentally measured values. Note that the uncertainties are added in quadrature ($63^2 + 61^2 = 88^2$).

If instead the Higgs particle H_μ is much heavier than the muon, its effect is negligible, which implies that the results no longer match each other:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = +0.000\ 000\ 003\ 02(88). \tag{49}$$

In other words, the experiment demonstrates that the Higgs particle contributing to a_μ is lighter than the muon. That is, $m_{H_\mu} < m_\mu = 105.66$ MeV$/c^2$.

6.4 The first particles

Our observations undeniably demonstrate that a material universe is possible. Therefore, the probability for a transition from literally nothing to a material universe cannot be exactly zero. Consequently, a transition must occur. Since no time exists in literally nothing, the spontaneous symmetry breaking must happen at the beginning of time.

The principle of maximum simplicity [4] suggests that as few symmetries as possible should be broken in the transition. That is, the universe should appear in the form of a single, spinless and neutral particle on which no forces act because no force-mediating gauge boson exists. Lacking the ability to emit and absorb photons, the particle is unable to multiply by forming virtual copies of itself in the way charged particles may do.

An ideal candidate for the first particle is described by Paul Dirac in his "new equation" of 1971 [13]. According to Biedenharn, Han, and van Dam [14], Dirac's neutral spinless particle represents a relativistic harmonic oscillator which can only exist alone (that is, in a universe void of charged particles with accompanying photons carrying electric force) at the same time as it may be viewed as "a realization on" two oppositely charged bosons. Therefore, the D particle may (in analogy with Schrödinger's cat) simultaneously be alive in its original massive shape and dead after annihilating into a pair of massless photons and in addition (unlike Schrödinger's cat with only one life) live a second life after decaying into a pair of massive charged bosons (which, in turn, annihilates into a massless pair of photons).

Viewing the D particle as an oscillator, it is natural to assume that it needs a certain, nonzero "time of creation" (t_c) to form, and will attempt to disappear within an equally long time; that is, have a lifetime of $\tau_D = t_c$.

6.5 The momentum equation

The momentum equation — also referred to as the fundamental hydrodynamical equation — describes the motion of fluids (liquids and gases). It derives from the law of conservation of momentum (Newton's second law of motion) and connects a fluid's velocity ($\boldsymbol{v}$) and pressure (p) to its density (ρ). For a nonviscous fluid [15],

$$\frac{\partial \boldsymbol{v}}{\partial t} - \boldsymbol{v} \times (\nabla \times \boldsymbol{v}) + \frac{1}{2}\nabla \boldsymbol{v}^2 + \frac{1}{\rho}\nabla p = 0 \tag{50}$$

holds in the absence of external forces.

For an ideal gas and an adiabatic process (a process in which heat does not enter or leave the system), $p\rho^{-\gamma} = p_0\rho_0^{-\gamma}$, where γ is a numerical constant [16]. That is, $\nabla p = p_0\rho_0^{-\gamma}\nabla\rho^{\gamma}$, from which follows that $\frac{1}{\rho}\nabla p = p_0\rho_0^{-\gamma}\rho^{-1}\nabla\rho^{\gamma} = p_0\rho_0^{-\gamma}\frac{\gamma}{\gamma-1}\nabla\rho^{\gamma-1}$ (since normal rules of derivation apply to the gradient: $\rho^{-1}\frac{\mathrm{d}}{\mathrm{d}x}\rho^{\gamma} = \frac{\gamma}{\gamma-1}\frac{\mathrm{d}}{\mathrm{d}x}\rho^{\gamma-1}$ because both expressions equal $\gamma\rho^{\gamma-2}\frac{\mathrm{d}}{\mathrm{d}x}\rho$). Thus, elimination of p from Eq. (50) yields

$$\frac{\partial \boldsymbol{v}}{\partial t} - \boldsymbol{v} \times (\nabla \times \boldsymbol{v}) + \nabla\left(\frac{1}{2}\boldsymbol{v}^2 + \frac{\gamma}{\gamma-1}p_0\rho_0^{-\gamma}\rho^{\gamma-1}\right) = 0. \tag{51}$$

Introducing the velocity of sound v_0 and the number of degrees of freedom f via $v_0^2 = \gamma p_0/\rho_0$ and $\gamma = 1 + 2/f$, which hold for an ideal gas [16, 17], Eq. (51) becomes

$$\frac{\partial \boldsymbol{v}}{\partial t} - \boldsymbol{v} \times (\nabla \times \boldsymbol{v}) + \frac{1}{2}v_0^2\,\nabla\left(\left(\frac{\boldsymbol{v}}{v_0}\right)^2 + f\left(\frac{\rho}{\rho_0}\right)^{2/f}\right) = 0, \tag{52}$$

which for a potential (that is, irrotational) flow,

$$\nabla \times \boldsymbol{v} = 0, \tag{53}$$

has the stationary ($\partial \boldsymbol{v}/\partial t = 0$) solution

$$\rho = \rho_0 \left(1 - \frac{1}{f}\frac{\boldsymbol{v}^2}{v_0^2}\right)^{f/2}, \tag{54}$$

where v_0 is the velocity of sound in the ideal gas and f the number of degrees of freedom of the gas molecules. The value of f is 3 for monatomic, 5 for diatomic, and 6 for polyatomic molecules (such as He, O_2, and CO_2, respectively).

Imagining that space in some respects may be compared to a physical fluid, one must assume that, unlike a gas, space cannot possess internal pressure (which would make it rapidly thin out unless the entire universe was contained in a kind of pressure cooker). Therefore, one may try to apply Eq. (54) to space, assuming it to be a kind of pressureless "space equation" describing a "whirl" in space. This stationary whirl may be assumed to show a "frozen" picture of an elementary particle (such as the electron) at the exact instant of its first appearance.

The density of a gas is defined as mass per volume ($\rho = m/V$), while its pressure results from molecules bouncing off each other. The fact that space must lack pressure suggests that it lacks molecules, too. Consequently, ρ should not be looked upon as mass (or number of molecules) per volume, but must be regarded as a fundamental, unobservable property of space. Also, the nonexistence of "space molecules" means absence of an intrinsic yardstick in space. In other words, space cannot be used as a coordinate system for determination of position, distance, or size.

Since the number of degrees of freedom appearing in the space equation (54) cannot characterize molecules, it must instead characterize the velocity $\boldsymbol{v}$ of the flow.

Thus, $f = 3$ should be associated with a spherically symmetric rotation that gives rise to the electric charge and rest energy (or mass) of charged spinless (μ_0 and τ_0) and spinning (e, μ, and τ) leptons. However, being physically impossible, the attempted spherically symmetric rotation of a newborn particle is never realized. Instead, preserving its energy content, electric charge, and (optional) spin angular momentum, the stationary particle immediately upon its formation disintegrates into a cloud of virtual photons.

Similarly, $f = 2$ apparently is linked to a cylindrically symmetric rotation responsible for the spin of the electron, muon, and tauon.

Finally, energy causes creation of space that leads to an essentially one-dimensional ($f = 1$) flow of space out from the particle. This effect, in turn, causes space to expand.

The force of gravitation, in turn, is a direct result of the expansion. As such, it cannot affect the overall expansion of the universe in any way. Therefore, it must "balance itself" by turning repulsive for very large cosmological distances.

From Eq. (54), two particle equations may be constructed. One of them,

$$\rho = \rho_0 \left(1 - \frac{v^2}{v_0^2}\right)^{1/2} \left(1 - \frac{1}{3}\frac{w^2}{w_0^2}\right)^{3/2}, \tag{55}$$

describes a spinless particle (a spin-0 muon or a spin-0 tauon). The other,

$$\rho = \rho_0 \left(1 - \frac{v^2}{v_0^2}\right)^{1/2} \left(1 - \frac{1}{2}\frac{u^2}{u_0^2}\right) \left(1 - \frac{1}{3}\frac{w^2}{w_0^2}\right)^{3/2}, \tag{56}$$

describes a spin-$\frac{1}{2}$ particle (electron, muon, or tauon). The velocities v, u, and w are associated with $f = 1$, 2, and 3, respectively. The two particle equations demonstrate that charge, spin, and expansion (with accompanying gravity) are intimately connected via the degrees-of-freedom parameter f.

The two equations provide instantaneous "static" pictures of the rematerialized particles. The static picture is immediately destroyed as dynamic interactions resume.

Since w (associated with $f = 3$) in the hydrodynamic lepton equation (56) is assumed to create energy, the energy of a single charged lepton may be written as

$$E_0 = m_0 c^2 = \frac{1}{2} \int\limits_V \rho w^2 \, \mathrm{d}V. \tag{57}$$

For the interaction energy between two electrons a distance a apart, one finds (see Appendix A.1)

$$E_{\mathrm{int}} = \pm 4\pi \rho_0 w_0^2 r_0^4 / a \tag{58}$$

when

$$w = \pm w_0 r_0^2 / r^2 \tag{59}$$

is assumed. In classical physics the interaction energy between two charged leptons is

$$E_{\mathrm{int}} = \pm e^2 / a \tag{60}$$

(setting $4\pi\epsilon_0 = 1$). Equating the hydrodynamic and the classical expressions for the interaction energy gives

$$4\pi \rho_0 w_0^2 r_0^4 = e^2, \tag{61}$$

which may be used to eliminate the unobservable parameter ρ_0.

Inserting Eqs. (59) and (61) into (57) and integrating over all space with real and positive ρ, one obtains the relation

$$r_0 m_0 c^2 / e^2 = B/2 \tag{62}$$

between the lepton's radius, mass, and charge. The lepton-structure constant characterizing this relation is

$$B = r_0 \int\limits_0^{\pi/2} \sin\theta \, \mathrm{d}\theta \int (\rho/\rho_0) \, \mathrm{d}r / r^2. \tag{63}$$

After ρ/ρ_0 is eliminated using Eq. (56), it is seen that the integration in Eq. (63) produces a well-defined numerical constant.

The integration extends down to $r = 3^{-1/4} r_0$, where ρ becomes zero. That is, a cosmological radius appears inside the particle. Therefore, the core of the charged lepton is not a string or knot or anything, but literally nothing — a hole in space. Consequently, the bare mass (m_0) of the particle is zero, in agreement with the JBW theory discussed in Sec. 6.1.

To calculate the lepton-structure constant B defined in Eq. (63), the fields u and v appearing in the hydrodynamic lepton equation (56) must first be specified. Field w, creating charge, is already given in Eq. (59).

It is assumed, see discussion following Eq. (56), that the field u creates spin. The simplest non-trivial solution to Eq. (53) is in polar coordinates

$$u = u_0 r_0 / r \sin\theta. \tag{64}$$

Since $r \sin\theta$ is the perpendicular distance from the z axis, Eq. (64) describes an essentially two-dimensional cylindrically symmetric rotation. For the angular momentum, $s = \int r \sin\theta \, u \, \mathrm{d}m$, of the particle, one obtains

$$s = m_0 r_0 u_0. \tag{65}$$

If the third field, the linear "velocity" v, is neglected, Eq. (63) may be directly integrated to yield as a first approximation (see Eq. (A.10) in Appendix A.2)

$$B_1 = 0.669\,605\,309\,417. \tag{66}$$

Since the space created per unit time is assumed to be proportional to the energy density (see discussion following Eq. (56)), the equation of continuity (the mathematical form of the law of conservation of mass) becomes

$$\nabla \cdot (\rho \boldsymbol{v}) = \rho_0 A \,\mathrm{d}E/\mathrm{d}V, \tag{67}$$

where a new parameter or constant of proportionality, A, is introduced. Equation (67) yields, via application of the divergence theorem,

$$m_0 c^2 = \int\limits_V \frac{\mathrm{d}E}{\mathrm{d}V}\,\mathrm{d}V = \frac{1}{A\rho_0}\int\limits_V \nabla \cdot (\rho\boldsymbol{v})\,\mathrm{d}V = \frac{1}{A\rho_0}\oint\limits_S (\rho\boldsymbol{v})\cdot \mathrm{d}\boldsymbol{S} = \frac{4\pi}{A}\lim_{r\to\infty} r^2 v, \tag{68}$$

where S is the surface of a sphere with radius r. The obvious assumptions, compare with Eqs. (59) and (64),

$$\lim_{r\to\infty} v = v_0 r_0^2/r^2 \tag{69}$$

and

$$v_0 = c \tag{70}$$

mean that A, instead of being an adjustable parameter, is determined by

$$A = 4\pi \frac{r_0^2}{m_0 c} \tag{71}$$

and thus eliminated from the calculations.

All three fields, w, u, and v, appearing in the lepton equation (56) are now known and the lepton-structure constant B may be computed. In Appendix A.2, it is shown how Eq. (67) may be transformed into an elliptic partial differential equation in two dimensions, with the help of which Eq. (63) may be iteratively integrated using B_1 of Eq. (66) as the initial approximation. The result is

$$B = 0.666\,001\,731\,498. \tag{72}$$

The constant B obtained from Eq. (56) assumably characterizes a charged spin-$\frac{1}{2}$ lepton. Similarly, the constant

$$B_0 = 0.978\,396\,401\,9 \tag{73}$$

obtained from Eq. (55) should characterize a "spinless lepton."

Next, one wants to know how the numerical constant B might be related to standard particle physics. Introducing the fine-structure constant

$$\alpha = e^2/\hbar c \tag{74}$$

(setting again $4\pi\epsilon_0 = 1$), one obtains from Eqs. (65) and (62)

$$s = B\alpha(u_0/c)\hbar/2 = \hbar/2, \tag{75}$$

since s must be equal to the lepton spin. Thus, the ratio u_0/c has the value $1/B\alpha = 205.759\,22$. This value is only 0.5 % less than the measured muon–electron mass ratio 206.768 28. Assuming tentatively that in the model the two ratios equal each other, one obtains

$$u_0/c = m_\mu/m_e \tag{76}$$

and

$$m_\mu/m_e = 1/B\alpha = 205.759\,22. \tag{77}$$

Further arguments [18] based on less rigorous mathematics lead to a gravitational potential,

$$U = -Gmr^{-1}(1 - r^2/R^2)^{-1}, \tag{78}$$

implying a repulsive gravitational force for distances $r > R/\sqrt{3}$, where $R = c/H$ is the radius of the universe defined as the velocity of light divided by the Hubble expansion rate. Another conclusion is that the density of the universe should be

$$\rho_u = 3H^2/4\pi G, \tag{79}$$

which is twice as high as its so-called critical density. In addition, the value

$$\frac{1}{H_0} = \frac{B^2(e^2/m_e c^2)^2 c}{32Gm_e} = 17.2\,\mathrm{Gyr} \tag{80}$$

or

$$H_0 = 56.8\ \mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{81}$$

for the present-day Hubble expansion rate is advocated.

6.6 Stationary and dynamic particles

Setting $\partial/\partial t = 0$ in the pressureless momentum equation, which is assumed to describe space (see Sec. 6.5), one obtains a stationary solution to the equation. This solution is assumed to describe a particle — either an electron via Eq. (56) or a spinless muon via Eq. (55) — at the exact instant of its materialization from radiation.

Since both the electron and the spinless muon lack bare mass (see Sec 6.1 and comment after Eq. (63) in Sec. 6.5), the law of conservation of mass is not applicable, but energy, charge, and spin angular momentum are conserved in the particle's transformation from a stationary structure to a dynamic "cloud" of virtual photons.

Thanks to the photon's lack of mass, the amount of electromagnetic matter it generates is perfectly flexible. Therefore, new particles (such as the Higgs and Z bosons) may later appear and contribute to the self-energy of the charged lepton — or any charged particle — while the total energy of the particle is being conserved through a corresponding decrease in photon energy.

The fact that the charged pion ($\pi^\pm$) has a larger self-energy (or mass) than the electron from which it is supposed to have emanated suggests that its formation proceeds in a manner that differs from the materialization of photons into electrons or spinless muons. The pion's formation may be thought to proceed as follows.

The last electrons are forced to pairwise "freeze" (that is, rematerialize instead of finishing their attempted annihilation into photons) with their charge split in two fractions and their spin cylinder bent into a circle, giving them a zero outward spin. Consequently, the stationary pion ($\underline{\pi}^\pm$) inherits the electron's self-energy and charge, but not its spin.

The stationary pion should be described by a solution to the pressureless momentum equation — either Eq. (52), or a generalized version of it expressed in more than three dimensions. Being physically inadmissible, it disintegrates into quarks ($\underline{\pi}^+ \to \mathrm{u\bar{d}}$, $\underline{\pi}^- \to \mathrm{d\bar{u}}$) — again with the particle's self-energy conserved.

At this point, the pion still has the same self-energy as the electron from which it originates. Therefore, it can only exist in a virtual state. (A virtual particle may exist for short moments Δt restricted by the Heisenberg uncertainty principle $\Delta E \times \Delta t \geq \frac{1}{2}\hbar$.)

To transform the pion into a physical, dynamically interacting particle, the quarks must get an additional energy amounting to $(m_\pi - m_e)c^2$. The purpose of the first Higgs boson is to obtain this energy from lepton pairs appearing in the propagators of the background photons and deliver it to the quarks (see Sec. 6.2).

A Derivations

A.1 The electric force

Let there be two particles, one at $z = 0$ and the other at $z = -a$. Suppose, first, that both give rise to cylindrically symmetric rotations with axes of rotation parallel to the x axis. Consider a point in the yz plane at a distance $\boldsymbol{r}_1$ from the first and $\boldsymbol{r}_2$ from the second particle (i.e., $\boldsymbol{r}_1 = \boldsymbol{a} + \boldsymbol{r}_2$). The square of the sum of the two velocities is

$$w^2 = (\boldsymbol{w}_1 + \boldsymbol{w}_2)^2 = (\boldsymbol{\omega}_1 \times \boldsymbol{r}_1 + \boldsymbol{\omega}_2 \times \boldsymbol{r}_2)^2. \tag{A.1}$$

If, instead, $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ represent the corresponding radial velocities, then

$$w^2 = (\boldsymbol{w}_1 + \boldsymbol{w}_2)^2 = (w_1 \boldsymbol{r}_1 / r_1 + (w_2 \boldsymbol{r}_2 / r_2)^2 \tag{A.2}$$

holds. Since the velocities of the first case are perpendicular to the velocities of the second case, the resulting w^2 is the same for the two cases.

Assume, therefore, that for the three-dimensional rotation (which is a mathematical generalization that cannot be visualized) the same correspondence holds, and that, consequently, Eq. (A.2) may be used in the calculations. With $w_i = \pm w_0 r_0^2 / r_i^2$ $(i = 1, 2)$ according to Eq. (59) and $\boldsymbol{r}_1 = \boldsymbol{r}$, Eq. (57) then yields for the interaction energy

$$\begin{aligned} E_{\text{int}} &= \pm\frac{1}{2}\int \rho \frac{2\, w_1 w_2}{r r_2} \boldsymbol{r} \cdot \boldsymbol{r}_2 \,\mathrm{d}V \\ &= \pm \int \rho \frac{w_0^2 r_0^4}{r^3 \mid \boldsymbol{r} - \boldsymbol{a} \mid^3} \boldsymbol{r} \cdot (\boldsymbol{r} - \boldsymbol{a}) \,\mathrm{d}V \\ &= \pm w_0^2 r_0^4 \rho_0 \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \sin\theta \,\mathrm{d}\theta \int_0^{\infty} \frac{r + a\cos\theta}{(r^2 + a^2 + 2ra\cos\theta)^{3/2}} \,\mathrm{d}r \\ &= \pm 4\pi \rho_0 w_0^2 r_0^4 / a, \end{aligned} \tag{A.3}$$

since $\rho = \rho_0$ may be assumed for $a \gg r_0$.

A.2 The constant B

Using Eqs. (57), (59), (61), (62), and (71), one obtains from Eq. (67),

$$B\,\nabla\cdot(\rho\boldsymbol{v}) = c\rho r_0^3/r^4. \tag{A.4}$$

Eq. (53) implies that

$$B\boldsymbol{v} = -cr_0\nabla f, \tag{A.5}$$

where f is a dimensionless scalar function chosen such that

$$\lim_{r\to\infty} f = Br_0/r \tag{A.6}$$

according to Eq. (69). Insertion of Eq. (A.5) in (A.4) yields

$$\nabla^2 f + \frac{1}{\rho}\nabla\rho\cdot\nabla f + \frac{r_0^2}{r^4} = 0. \tag{A.7}$$

Using Eqs. (56), (59), (64), and (A.5), ρ/ρ_0 is expressed as a function of f and the polar coordinates r and θ. In a coordinate system with $\alpha = 2^{-1/2}r_0/r$ and $\beta = \cos\theta$, the elliptic partial differential equation (A.7) may be written as

$$af_{\alpha\alpha} + cf_{\beta\beta} + ef_\alpha + gf_\beta = p + d, \tag{A.8}$$

where

$$\begin{aligned}
&a = \alpha^2 \quad\rightarrow\quad \frac{5}{2}\alpha^2 \;\text{ for } \alpha = (3/4)^{1/4},\\
&c = 1-\beta^2,\\
&e = -8\alpha^5\left(1-\frac{4}{3}\alpha^4\right)^{-1} - 2\alpha^3\left(1-\beta^2-\alpha^2\right)^{-1} \quad\rightarrow\quad 0 \;\text{ for } \alpha = (3/4)^{1/4},\\
&g = -2\beta - 2\alpha^2\beta\left(1-\beta^2-\alpha^2\right)^{-1},\\
&p = -2\alpha^2, \;\text{ and}\\
&d = 2B^{-2}\alpha^2\left[1 - 2B^{-2}\alpha^2\left(\alpha^2 f_\alpha^2 + (1-\beta^2)f_\beta^2\right)\right]^{-1}\\
&\quad\times\left[2\alpha^3 f_\alpha^3 + (1-\beta^2)f_\beta^2(\alpha f_\alpha - \beta f_\beta) + \alpha^4 f_\alpha^2 f_{\alpha\alpha}\right.\\
&\quad\left. + 2\alpha^2(1-\beta^2)f_\alpha f_\beta f_{\alpha\beta} + (1-\beta^2)^2 f_\beta^2 f_{\beta\beta}\right]
\end{aligned}$$

and the boundaries (see Fig. A.1) and boundary conditions are

$$\begin{aligned}
\alpha = 0:&\quad f = \text{ constant (e.g., } f = 0);\\
\beta = 0:&\quad f_\beta = 0;\\
\alpha = (3/4)^{1/4}:&\quad f_\alpha = 0;\\
\alpha^2+\beta^2 = 1:&\quad \alpha f_\alpha + \beta f_\beta = 0.
\end{aligned}$$

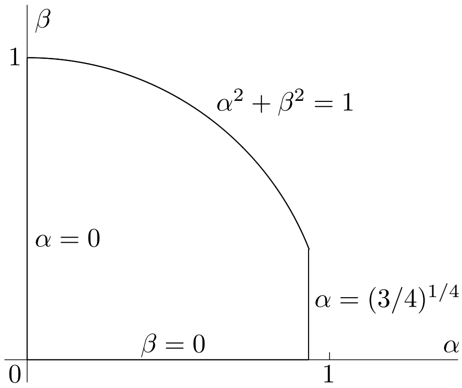


Figure A.1: Boundaries of Eq. (A.8).

The differential equation is integrated numerically by placing a rectangular grid over the area within the boundary, defining a linear equation for each mesh point, and solving the resulting matrix equation [19]. External points near the curved boundary are related to the internal points through the normal derivative.

In Step 1, the matrix is factorized and the equation solved for the vector f, assuming $d = 0$. Then, B is found from [compare with Eq. (63)]

$$B = B_1 - D, \tag{A.9}$$

where

$$\begin{aligned} B_1 &= 2^{1/2} \int_0^{(\frac{3}{4})^{1/4}} \left(1 - \frac{4}{3}\alpha^4\right)^{3/2} \mathrm{d}\alpha \int_0^{(1-\alpha^2)^{1/2}} \left(1 - \frac{\alpha^2}{1-\beta^2}\right) \mathrm{d}\beta \\ &= 0.669\,605\,309\,417\,211 \end{aligned} \tag{A.10}$$

and

$$D = 2^{1/2} B^{-2} \int_0^{(\frac{3}{4})^{1/4}} \left(1 - \frac{4}{3}\alpha^4\right)^{3/2} \mathrm{d}\alpha \int_0^{(1-\alpha^2)^{1/2}} \left(1 - \frac{\alpha^2}{1-\beta^2}\right) H(\alpha,\beta)\, \mathrm{d}\beta, \tag{A.11}$$

with

$$H(\alpha,\beta) = (r_0 \nabla f)^2 \left(1 + \left[1 - B^{-2}(r_0\nabla f)^2\right]^{1/2}\right)^{-1}, \tag{A.12}$$

in which

$$(r_0\nabla f)^2 = 2\alpha^4 f_\alpha^2 + 2\alpha^2(1-\beta^2) f_\beta^2. \tag{A.13}$$

In Step 2, the vector d is calculated using the previously obtained values for f and B. Using the new d, the matrix equation is again solved for f, and an improved value is calculated for B. This step is repeated until the desired precision has been attained.

The results are better behaved if a rectangular boundary is used. Such a boundary may be obtained by introducing the new variables $x = \alpha$ and $y = \beta(1-\alpha^2)^{-1/2}$. Computation of B using an increasing mesh density now leads to a series of values converging toward the value in Eq. (72).

A.3 QED correction to the muon mass

The spinning leptons of phase 3 are the same as today's electron ($e^{\pm}$), muon ($\mu^{\pm}$), and tauon ($\tau^{\pm}$). Their interaction with the photon (γ) is described by the one-photon vertex of spinor QED. The interaction between the spinless muon ($\mu_0^{\pm}$) of phase 2 and its corresponding photon (γ_μ) is described by scalar QED. In the Feynman graphs of scalar QED, both one-photon and two-photon vertices appear [20].

The photon is a spin-1 particle that may (or may not) have an orbital angular momentum that cancels out its spin. Being end products of spinless-muon pair annihilation, the real photons of phase 2 form outward spinless pairs. Simplicity suggests that the individual photon possesses a nonzero total angular momentum identical with its spin. Since angular momentum is a conserved quantity in physics, such a photon cannot disintegrate into a pair of spinless muons. It can only form spinless-muon loops via the two-photon vertex of scalar QED.

Fig. A.2 illustrates the third phase transition, when the phase-2 photons (γ_μ) materialize as pairs of spinning electrons in phase 3. The vertical line indicates the instant of transition.

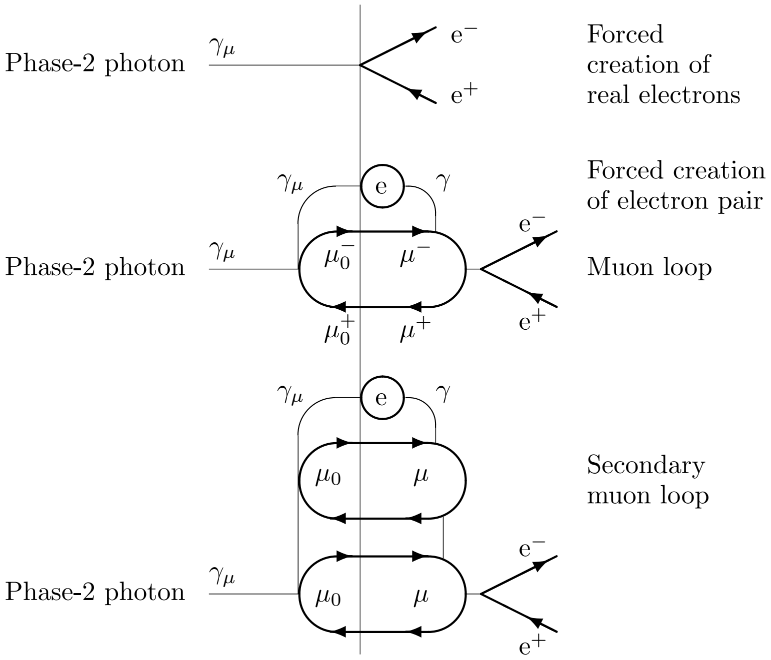


Figure A.2: The third phase transition.

In the first case (top of Fig. A.2), the real phase-2 photon materializes into an electron and a positron.

In the second case, the phase-2 photon emits a muon pair. The muons acquire spin and an initial natural mass m_μ. But instead of recapturing the pair of spinless muons it emitted in phase 2, the photon itself is, after materializing into an electron pair that again annihilates into a photon, captured by the spinning muon of phase 3, thus adding the mass $2m_e$ (created in the materialization of the photon) to the mass of the muon pair. Therefore, the muon pair possesses a mass of $2m_\mu + 2m_e$ when it annihilates forming a real phase-3 photon that materializes into a pair of real electrons.

In the third case (bottom of Fig. A.2), the photon emits two muon pairs in phase 2, and is itself absorbed by the phase-3 muon of the secondary muon loop. The materialized mass, $2m_e$, affects the final mass of the primary (bottom) muon pair in proportion m_e to m_μ, since it is transferred to it via the secondary muon loop (with associated mass $2m_\mu$) ending in a phase-3 photon (with associated mass $2m_e$). Therefore, the diagram's contribution to the mass of the bottom muon pair is $(m_e/m_\mu)2m_e$.

Obviously, any number of muon loops may appear in diagrams of the type described. In the path-integral formulation of quantum theory, a particle simultaneously takes all possible paths. Adding, therefore, the contributions from all diagrams, one obtains $2m_\mu + 2m_e + (m_e/m_\mu)2m_e + (m_e/m_\mu)^2 2m_e + \cdots$ for the mass of the first muon pair. Note that the fine-structure constant α, being essentially a measure of the electron–muon mass ratio, is not defined for phase 2. Therefore, the natural phase-2 probability amplitude is 1.

For each secondary loop running clockwise (like the one in Fig. A.2, third case), there is another loop running counterclockwise. Consequently, the contribution to the mass of the primary muon pair from all diagrams is $2\bar{m}_\mu = 2m_\mu + 2m_e + 2(m_e/m_\mu)\,2m_e + 2^2(m_e/m_\mu)^2\,2m_e + \cdots$, and the mass of one muon becomes

$$\begin{aligned}\bar{m}_\mu &= m_\mu + m_e + 2(m_e/m_\mu)m_e + 2^2(m_e/m_\mu)^2 m_e + 2^3(m_e/m_\mu)^3 m_e + \cdots \\ &= m_\mu + m_e/(1 - 2m_e/m_\mu),\end{aligned} \tag{A.14}$$

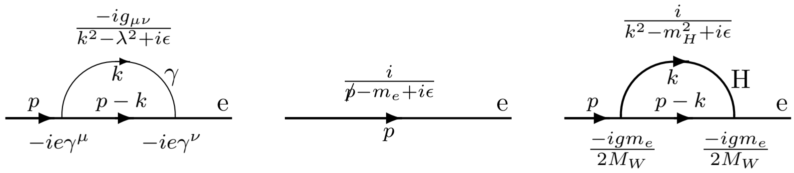
which is equivalent to

$$\begin{aligned}\bar{m}_\mu/m_e &= 1/B\alpha + 1 + 2B\alpha + 4(B\alpha)^2 + 8(B\alpha)^3 + \cdots \\ &= 1/B\alpha + 1/(1 - 2B\alpha) \\ &= 206.769\,04,\end{aligned} \tag{A.15}$$

where $\bar{m}_\mu$ is the QED-corrected muon mass.

A.4 Higgs contribution to the lepton mass

In the figure, the propagators for the photon, electron, and Higgs are shown above their corresponding particle lines, while the expressions for the photon–electron and Higgs–electron vertices are shown below the electron line:



The notation follows the convention established by James Bjorken and Sidney Drell in their book *Relativistic Quantum Mechanics* [21].

Thus, in Feynman's slash notation, $\not p$ is the inner product of the four vector γ and the four momentum p, or

$$\not p = \gamma \cdot p = \gamma^\mu p_\mu = \gamma_\mu p^\mu, \tag{A.16}$$

where the convention of summing over repeated indices is used (e.g., $\gamma^\mu p_\mu = \gamma^0 p_0 + \gamma^1 p_1 + \gamma^2 p_2 + \gamma^3 p_3$). For the time component of a four vector such as p, it holds that $p^0 = p_0$, and for its space components, $p^i = -p_i$ ($i = 1,\ 2,\ 3$). The components p_1, p_2, and p_3 form the momentum vector $\boldsymbol{p}$. The same rules apply to the four vector γ ($\gamma^0 = \gamma_0$ and $\gamma^i = -\gamma_i$ with $(\gamma_1,\ \gamma_2,\ \gamma_3) = \boldsymbol{\gamma}$).

The arrows shown in the figure indicate four momentum — p for the electron, and k for the photon and Higgs. The indices μ and ν indicate that summation over photon and electron polarizations must be performed for the photon–electron loop, while no similar summation is needed for the Higgs–electron loop (since the scalar Higgs boson lacks polarization). For computational reasons, the photon is attributed an infinitesimal mass (λ) that is set equal to zero in final results.

Moving clockwise around the loops and multiplying the expressions with each other, one obtains for the integrand associated with the left (photon–electron) loop,

$$I(\gamma) = \frac{-ig_{\mu\nu}}{k^2 - \lambda^2 + i\epsilon} (-ie\gamma^\nu) \frac{i}{\not p - \not k - m_e + i\epsilon} (-ie\gamma^\mu) \tag{A.17}$$

and for the integrand associated with the right (Higgs–electron) loop,

$$I(\mathrm{H}) = \frac{i}{k^2 - m_H^2 + i\epsilon} \left(-ig\frac{m_e}{2M_W}\right) \frac{i}{\not p - \not k - m_e + i\epsilon} \left(-ig\frac{m_e}{2M_W}\right). \tag{A.18}$$

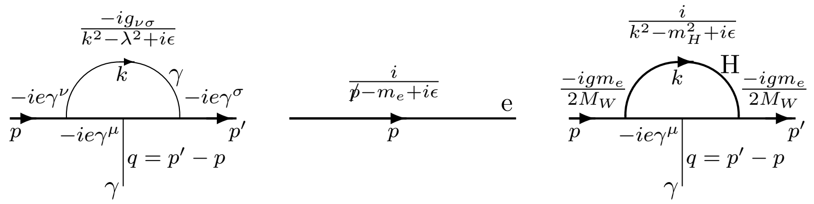
The symbol $g_{\mu\nu}$ appearing in the photon propagator is a 4×4 matrix given in Ref. [21], on p. 281, and the components of the four vector γ are the Dirac matrices shown on p. 282.

Following the text book, and handling the two integrals in parallel, one finds that in the limit when $k \to \infty$ (and the integrals diverge), the ratio between the two integrands is

$$\frac{I(\mathrm{H})}{I(\gamma)} = \frac{G_F m_e^2}{4\sqrt{2}\pi\alpha}. \tag{A.19}$$

A.5 Higgs contribution to the lepton $g-2$

On pages 166–172 in their text book [21], Bjorken and Drell calculate the part, $a_e^{(2)}(\gamma)$, of the electron anomalous magnetic moment that derives from the figure's left diagram:



The diagram consists of a self-mass loop with an external photon line added. The photon mediates the force between the external magnetic field and the electron. Its momentum q is the difference between the electron's final momentum p' and its initial momentum p.

The expression obtained from the Feynman rules may be written as $-ie\Lambda_\mu(p',p)$, where $\Lambda_\mu(p',p)$ is given by

$$\begin{aligned}
\Lambda_\mu(p',p) &= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{-ig_{\nu\sigma}}{k^2-\lambda^2+i\epsilon}(-ie\gamma^\sigma)\frac{i}{\not{p}'-\not{k}-m_e+i\epsilon}\gamma_\mu\frac{i}{\not{p}-\not{k}-m_e+i\epsilon}(-ie\gamma^\nu) \\
&= (-ie)^2\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{-i}{k^2-\lambda^2+i\epsilon}\gamma_\nu\frac{i}{\not{p}'-\not{k}-m_e+i\epsilon}\gamma_\mu\frac{i}{\not{p}-\not{k}-m_e+i\epsilon}\gamma^\nu \\
&= (-ie)^2\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-\lambda^2}\frac{\gamma_\nu(\not{p}'-\not{k}+m_e)\gamma_\mu(\not{p}-\not{k}+m_e)\gamma^\nu}{((p'-k)^2-m_e^2)((p-k)^2-m_e^2)}.
\end{aligned} \tag{A.20}$$

See Eq. (8.49) on page 166 in the text book. For the corresponding Higgs contribution (right diagram), one similarly obtains

$$\begin{aligned}
\Lambda_\mu^{\mathrm{H}}(p',p) &= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m_H^2+i\epsilon}\frac{-igm_e}{2M_W}\frac{i}{\not{p}'-\not{k}-m_e+i\epsilon}\gamma_\mu\frac{i}{\not{p}-\not{k}-m_e+i\epsilon}\frac{-igm_e}{2M_W} \\
&= \left(\frac{-igm_e}{2M_W}\right)^2\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{-i}{k^2-m_H^2}\frac{(\not{p}'-\not{k}+m_e)\gamma_\mu(\not{p}-\not{k}+m_e)}{((p'-k)^2-m_e^2)((p-k)^2-m_e^2)}.
\end{aligned} \tag{A.21}$$

Because of the similarity between the expressions, the Higgs contribution may be found via a parallel calculation of the two cases in which only a small part of the calculation has to be redone for the Higgs.

A comparison between the two results gives for the respective second-order correction

$$\begin{aligned}
a_e^{(2)}(\gamma) &= \frac{\alpha}{2\pi}, \\
a_e^{(2)}(\mathrm{H}) &= \frac{3\sqrt{2}G_F m^2}{16\pi^2} = \frac{3G_F m^2}{8\sqrt{2}\pi^2},
\end{aligned} \tag{A.22}$$

where the correction to a_e is given in Eq. (8.64) in the text book.

The Higgs correction obtained in Eq. (A.22) holds for $m_H \ll m$. From the general expression [22] valid for all m_H, it is seen that $a_e^{(2)}(\mathrm{H}) = 0$ when $m_H \gg m$.

References

[1] P. A. M. Dirac, *Nature* **139** (1937) 323.

[2] P. A. M. Dirac, *Proc. R. Soc. Lond. A* **165** (1938) 199.

[3] P. A. M. Dirac, *Proc. R. Soc. Lond. A* **365** (1979) 19.

[4] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965) pp. 94–96.

[5] P. A. M. Dirac, *Proc. Camb. Phil. Soc.* **26** (1930) 361.

[6] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, 1980).

[7] S. Sundman, *Fortran program simulating the evolution of the universe*, http://www.physicsideas.com/Simulation.for.

[8] W. Greiner, S. Schramm, and E. Stein, *Quantum Chromodynamics*, 2nd edn. (Springer, 2002), pp. 150–151.

[9] R. Pohl *et al.*, *Nature* **466** (2010) 213.

[10] K. Johnson, M. Baker, and R. Willey, *Phys. Rev.* **136** (1964) B1111.

[11] M. Veltman, *Diagrammatica: The Path to Feynman Diagrams* (Cambridge University Press, 1994), pp. 270–271.

[12] M. Passera, W. J. Marciano, and A. Sirlin, *Phys. Rev. D* **78** (2008) 013009.

[13] P. A. M. Dirac, *Proc. R. Soc. Lond. A* **322** (1971) 435.

[14] L. C. Biedenharn, M. Y. Han, and H. van Dam, *Phys. Rev. D* **8** (1973) 1735.

[15] G. Joos, *Theoretical Physics*, 3rd edn. (Blackie & Son, London, 1959), p. 197.

[16] G. Joos, *Theoretical Physics*, 3rd edn. (Blackie & Son, London, 1959), p. 213.

[17] A. Sommerfeld, *Thermodynamics and Statistical Mechanics* (Academic Press, New York, 1956), p. 18.

[18] S. Sundman, *A simple model describing a pure QED universe*, http://www.physicsideas.com/Paper.pdf, p. 12f.

[19] L. Fox, *Numerical Solutions of Ordinary and Partial Differential Equations* (Pergamon Press, Oxford, 1962).

[20] W. Greiner and J. Reinhardt, *Quantum Electrodynamics*, 2nd edn. (Springer, 1996), p. 393.

[21] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[22] T. Kinoshita and W. J. Marciano, in *Quantum Electrodynamics*, ed. T. Kinoshita (World Scientific, Singapore, 1990), p. 463.

Index