Higgs demystified

October 4, 2010

1 The JBW model and Higgs corrections to lepton masses

According to basic quantum electrodynamics (QED) theory, the mass m of an electron is given by

$$m = m_0 + \delta m, \tag{1.1}$$

where the electron's bare mass m_0 is undetermined and its self-mass $\delta m = \delta m^{(2)} + \delta m^{(4)} + \cdots$ forms a series in powers of α in which all terms are infinite.

As a rule, corrections to measurable physical quantities are theoretically calculable in QED. For instance, the electron's anomalous magnetic moment a_e corrects the reduced gyromagnetic ratio g of the electron, yielding $g/2 = 1 + a_e = 1.001\ 159\ 652\ 18$. Exceptions to the rule are measurable mass ratios such as $m_\mu/m_e = 206.768\ 28$. Due to the bad behavior of Eq. (1.1), corrections to them are incalculable in basic QED.

However, in the early 1960s, Kenneth Johnson, Marshal Baker, and Raymond Willey developed a perturbation theory "within the usual formalism of quantum electrodynamics" [1] in which m_0 is zero and which yields finite values for the terms in the series for δm. Consequently, according to this so-called JBW hypothesis,

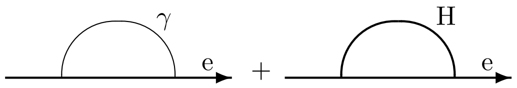
$$m = \delta m, \tag{1.2}$$

implying that "the electron mass must be totally dynamical in origin." In other words, the electron's rest energy mc^2 originates from virtual photons that form the electron.

Sometimes the JBW theory is referred to as "finite QED" because it rids QED of its divergences—making the infinite so-called renormalization constants Z_1, Z_2, Z_3, and δm finite. Another name, "pure QED," alludes to the fact that the JBW hypothesis turns QED into a consistent, self-contained theory. The JBW theory is rarely discussed in text books. For instance, in *Quantum Field Theory*, Claude Itzykson and Jean-Bernard Zuber only write (last lines on page 424): "The possibility of a finite quantum electrodynamics has been studied by K. Johnson, R. Willey, and M. Baker, *Phys. Rev.*, vol. 163, p. 1699, 1967, and S. L. Adler, *Phys. Rev.*, ser. D, vol. 5, p. 3021, 1972."

By replacing the badly behaving Eq. (1.1) with the well-behaved Eq. (1.2), the JBW theory extends the scope of the standard model (SM), turning basic SM into a kind of extended SM (xSM) theory in which corrections to particle masses should be calculable.

The figure shows the lowest-order (i.e., one-loop, two-vertex, or second-order) electron self-mass diagrams involving a virtual photon and a virtual Higgs, respectively.



Ignoring all other contributions to the electron mass, and using standard electroweak perturbation theory, one obtains from Eq. (1.2) the expression (see Section 4)

$$m = \delta m^{(2)}(\gamma) + \delta m^{(2)}(\mathrm{H}) = \ln\frac{\Lambda}{m}\left(\frac{3\alpha}{2\pi} + \frac{3G_F m^2}{8\sqrt{2}\pi^2}\right)m, \tag{1.3}$$

for the renormalized (i.e., physical) mass m of the electron. Here, Λ is an ultraviolet (UV) cutoff mass introduced to make the mathematics finite. Finite terms of order unity are not shown, since they are negligible in comparison with the leading, divergent terms.

Partly, the JBW theory is based on a non-perturbative approach to QED. In standard perturbative QED, the theory implies that the expression in the parenthesis of Eq. (1.3) should tend to zero when sufficiently many higher-order terms are added to the series forming m. Unfortunately, this prediction is impossible to verify; because of the explosion in computational complexity with increasing order that characterizes perturbative QED, only the first few terms in the series are computable.

However, this fact does not invalidate the assumption that the ratio between the Higgs and photon contributions to the electron mass may be inferred from the leading terms obtained from standard perturbation theory—that is, from the two terms in Eq. (1.3)—and that, consequently,

$$\frac{m(\mathrm{H})}{m(\gamma)} = \frac{G_F m^2}{4\sqrt{2}\pi\alpha} = 2.348\ 476(20) \times 10^{-11} \tag{1.4}$$

when the values $G_F/(\hbar c)^3 = 1.166\ 37(1){\times}10^{-11}\ \mathrm{MeV}^{-2}$ and $mc^2 = 0.510\ 998\ 910(13)$ MeV are used (reintroducing c and $\hbar$, which commonly are set equal to 1 in equations).

The value obtained in Eq. (1.4) suggests that corrections from Higgs and other weak loops to m are so small compared with the main contribution deriving from the photon loop that $m(\gamma)$ may be set equal to m.

Since the split-up of the lepton mass into the two components indicated by Eq. (1.3) is due to the existence of the Higgs, the splitting must have taken place at the instant the Higgs particle was born. Therefore, it is natural to assume that the original Higgs boson was delivered by the lepton, and that the purpose of the mass splitting was to furnish the Higgs with a mass of $m_H = m(\mathrm{H})$. In other words, xSM predicts

$$m_H/m = G_F m^2/4\sqrt{2}\pi\alpha = 2.348\ 476(20) \times 10^{-11} \tag{1.5}$$

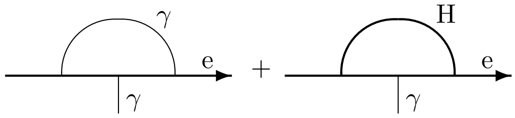
for the Higgs-to-electron mass ratio.

From Eq. (1.5), one obtains for the "flyweight" Higgs particle a rest energy of $m_H c^2 = 12.000\ 69(10)\ \mu$eV corresponding to the energy of a 2.9018 GHz microwave photon. Since the Higgs boson is a weakly interacting particle, it is rarely emitted by electrons and may, therefore, easily pass unnoticed in physical experiments. Also, because it has zero spin (while the photon is a polarized, spin-1 boson), it interacts with electrons in the same way as a photon lacking polarization would do. This similarity between Higgs and photon suggests that the Higgs may be difficult to distinguish from a photon.

Still, as suggested in http://www.physicsideas.com/Experiments.pdf, the flyweight Higgs particle should reveal itself in a variety of ways. For instance, its signature might be seen in the anomalous magnetic moment of the muon and in astrophysical observations.

2 Higgs corrections to the lepton $g - 2$

The figure shows the lowest, second-order (or one-loop) photon and Higgs contributions to the electron's anomalous magnetic moment, a_e.



Theoretically, these contributions are well-known. In an article in Ref. [2] discussing QED, hadronic (i.e., strong), weak, and possible "new physics" contributions to the muon anomalous magnetic moment, Toichiro Kinoshita and William J. Marciano present the second-order Higgs contribution to a_μ. See Eq. (5.4) on page 463 in their article. For an electrically charged lepton (electron, muon, or tauon) of mass $m \gg m_H$ (implying $F(m_H^2/m^2) = F(0) = 1$ as noted on page 464 in the article), the equation simplifies to

$$a^{(2)}(\mathrm{H}) = \frac{3G_F m^2}{8\sqrt{2}\pi^2}. \tag{2.1}$$

The sum of the second-order photon and Higgs contributions to a_e is

$$\begin{aligned} a_e^{(2)}(\gamma) + a_e^{(2)}(\mathrm{H}) = \alpha/2\pi + 3G_F m_e^2/8\sqrt{2}\pi^2 &= 0.001\ 161\ 409\ 732\ 89(43) \\ &+ 0.000\ 000\ 000\ 000\ 08, \end{aligned} \tag{2.2}$$

while the corresponding sum for a_μ is (using 105.658 369(9) MeV for $m_\mu c^2$)

$$\begin{aligned} a_\mu^{(2)}(\gamma) + a_\mu^{(2)}(\mathrm{H}) = \alpha/2\pi + 3G_F m_\mu^2/8\sqrt{2}\pi^2 &= 0.001\ 161\ 409\ 73 \\ &+ 0.000\ 000\ 003\ 50. \end{aligned} \tag{2.3}$$

The value used for α in the first term of Eq. (2.2) is theoretically obtained from the experimental value [3]

$$a_e^{\mathrm{EXP}} = 0.001\ 159\ 652\ 180\ 73(28), \tag{2.4}$$

and is more precise than the directly measured α value. Since there is no very accurate experimental value for α, there also is no precise theoretical a_e value with which a_e^{EXP} might be compared. Therefore, the only practical implication of the Higgs correction in Eq. (2.2) is that the theoretical α value obtained from Eq. (2.4) changes from [3]

$$\alpha^{-1}(a_e) = 137.035\ 999\ 084(51) \tag{2.5}$$

to

$$\alpha^{-1}(a_e) = 137.035\ 999\ 074(51). \tag{2.6}$$

In an article discussing "the muon $g - 2$ and the bounds on the Higgs boson mass" [4], Massimo Passera, William J. Marciano, and Alberto Sirlin obtain

$$a_\mu^{\mathrm{SM}} = 0.001\ 165\ 917\ 78(61) \tag{2.7}$$

for the theoretical a_μ value, and cite

$$a_\mu^{\mathrm{EXP}} = 0.001\ 165\ 920\ 80(63) \tag{2.8}$$

for the corresponding experimental value measured by the E821 experiment at Brookhaven. Clearly, the relatively large difference,

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = +0.000\ 000\ 003\ 02(88), \tag{2.9}$$

indicates a mismatch between theoretical prediction and physical reality.

However, the theoretical prediction in Eq. (2.7) was obtained under the assumption that $m_H \gg m$, which means that the Higgs contribution is negligibly small ($F(m_H^2/m_\mu^2) = 0$ in Eq. (5.4) on page 463 in Ref. [2]). The assumption that $m_H \ll m$ implies that the Higgs correction in Eq. (2.3) has to be added to the value in Eq. (2.7), yielding

$$a_\mu^{\rm xSM} = 0.001\ 165\ 921\ 28(61) \tag{2.10}$$

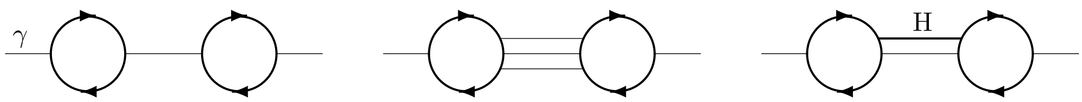
for the theoretical a_μ value, and a difference,

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm xSM} = -0.000\ 000\ 000\ 48(88), \tag{2.11}$$

indicating good agreement between the theoretically predicted and experimentally measured values.

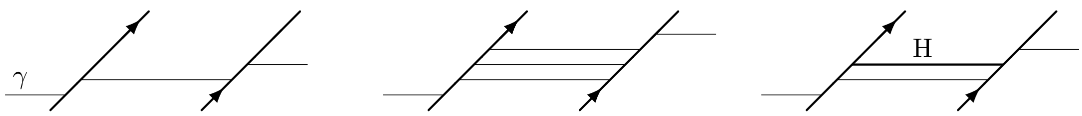
3 Higgs in astrophysics—the Pioneer anomaly

A free photon oscillates between various states. In addition to forming a single-photon state, it may form a short-lived polarization loop (that is, a particle pair consisting of a charged particle and its antiparticle). Also, it may for a brief moment form a photon triplet or a photon-Higgs pair (but, according to Furry's theorem, not a pair of photons):



Higgs bosons or other virtual massive particles appearing in the photon propagator do not slow down the speed of a free photon, which on average travels with the speed of light, c.

This picture changes if the polarization loops are replaced by real particles that are sufficiently near each other to allow exchange of virtual particles between them:



The time delay between absorption and reemission of a photon by a real particle (electron or proton, say) has the effect that light rays travel with speeds less than c in transparent media. The difference between the time a Higgs remains absorbed in a particle (right) and the time an extra photon pair does so (middle) introduces an anomaly in the signal speed. This anomaly is probably too small to be experimentally detectable.

Another anomaly results from the fact that the massive Higgs particle is slower than the massless photon, which always travels at speed c if one ignores quantum fluctuations that are smoothed out in macroscopic time-of-flight measurements. (See Richard P. Feynman, *QED: The Strange Theory of Light and Matter*, p. 89.)

When the energy of the photon is in the vicinity of the Higgs rest energy, the virtual Higgs particle that occasionally accompanies the massless photon in its jump between two real particles may be only slightly off mass-shell and have an energy that approximately equals its rest energy. When this happens, the Higgs has low kinetic energy and moves with a speed that may be much lower than c, thus causing an anomalous signal delay. This "Higgs delay" should be observable for photons that traverse vast regions of hydrogen (proton-electron) plasma.

Using the Fermi Gamma-ray Telescope to study the arrival-time-versus-frequency distribution of individual photons in gamma-ray bursts between 30 and 300 GeV, it should be possible to detect the existence of very heavy Higgs particles—even much heavier than 300 GeV because of the redshift of light arriving from very distant objects—and estimate its mass.

Similarly, if there exist Higgs particles of mass 12 $\mu\text{eV}/c^2$ corresponding to a microwave photon of frequency 2.9018 GHz, it might be possible to observe the Higgs delay of UHF radio signals passing through the solar-wind plasma.

Indeed, microwave radio signals are used for communication with spacecraft. Thus, the Pioneer 10/11 communication systems operated at frequencies of approximately 2.11 and 2.29 GHz (see *Phys. Rev.* D 65, 082004 (2002) pp. 4-5)—only some 30 percent below the predicted 2.9018 GHz threshold value for production of real Higgs particles.

An unrecognized Higgs delay causes signals to travel slightly slower than they are believed to do. Consequently, when a spacecraft reaches a region where the plasma is too thin to appreciably slow down the photon ray, the signal speed has increased more than estimated. Therefore, response signals from very distant spacecraft return earlier than expected. Which, in turn, inevitably leads to the conclusion that the distant spacecraft are closer to the sun than they actually are. Or that "Pioneer 10 is experiencing a mysterious deceleration towards the Sun" (*New Scientist*, 20 July 2002, p. 28).

4 Higgs contribution to the lepton mass

In Appendix A, Eq. (1.3) is derived from first principles. The calculation shows that the negative value obtained in Paper.pdf, Appendix C is wrong: according to SM, the Higgs contribution to the lepton mass is positive, not negative.

The first-obtained, erroneous minus sign led me to a specific electroweak model with the so-called "Higgs mechanism" replaced by a "Higgs-neutrino mechanism" (or, more precisely, "H–Z^0–$\phi^0\phi^+\phi^-\nu\text{W}^+\text{W}^-$ mechanism") in which the Higgs, Z boson, Higgs ghosts, neutrino, and W bosons are assigned well-defined roles and the Higgs and neutrino are loaded with precisely specified masses. Because this model results in several verifiable predictions, it cannot lightly be dismissed as unfounded.

So, could it be that in reality the Higgs contribution is negative? Technically, it is very easy to introduce a minus sign. It suffices to change the sign of the Higgs propagator appearing in the Feynman diagram.

However, modifying the Feynman rules of SM in this way would make the Higgs contribution to the lepton anomalous magnetic moment negative, too. This conclusion is confirmed by a direct calculation showing that $a^{(2)}(\text{H})$ in Eq. (2.1) takes on the sign of the Higgs propagator. See Appendix B.

Thus, modifying the Higgs propagator in order to obtain a negative value for $m(\text{H})$ would double the discrepancy in Eq. (2.9) to about 5 ppm instead of eliminating it. In view of the good match between the measured and theoretical muon-electron mass ratios (206.768 2823(52) and 206.768 2832(1), respectively, see http://www.physicsideas.com) with 0.025 ppm experimental uncertainty, it seems implausible that any "new physics" effects might cause a 5 ppm correction to the muon anomalous magnetic moment.

Therefore, the conclusion must be that instead of attempting to modify SM, one should reinterpret the Higgs-neutrino mechanism. This is easily done, since it turns out that the sign of $m(\mathrm{H})$ is of no relevance to the overall picture of the mechanism. Compare with the discussion preceding Eq. (1.5) on page 2.

5 The Higgs-neutrino mechanism

After the primordial (massive, neutral, and spinless) Dirac particle has popped up in a finite spacetime bubble and broken the perfect symmetry of literally nothing, the unstable matter content of the expanding universe undergoes repeated symmetry-breaking transformations until a stable proton-electron pair (pe^-) finally appears when the universe is about 4×10^{-15} seconds old:

$\mathrm{D} \rightarrow$	$\tau_0^+\tau_0^- \rightarrow$	$\underline{\mu}_0^+\underline{\mu}_0^- \rightarrow$	$\mu_0^+\mu_0^- \rightarrow$	$\underline{\mathrm{e}}^+\underline{\mathrm{e}}^- \rightarrow$	$\mathrm{e}^+\mathrm{e}^- \rightarrow$	Higgs-neutrino mechanism $\rightarrow$	$\mathrm{p\bar{p}} \rightarrow$	pe^-
1	9	0	23	0	37 293	1000	0.0	∞
1	10^2	10^2	10^3	2×10^3	3×10^9			

The underlined symbols indicate newborn, "frozen" particles that immediately turn into dynamically interacting particles. The first row of numbers indicates time elapsed during each stage of the evolution of matter in units of $t_c \approx 10^{-19}$ s. The second row of numbers displays the quantity of particles within each phase; hence, phase 1 begins with one D particle and ends with (about) 100 photon pairs resulting from tauon-pair annihilation; phase 2 begins with 100 spinless-muon pairs and ends with (about) 1000 photon pairs; and phase 3 begins with 2000 electron pairs (rematerialized pairwise from pairs of photons, $\gamma\gamma \rightarrow \mathrm{e}^+\mathrm{e}^-\ \mathrm{e}^+\mathrm{e}^-$) and ends with about 3 billion background photons.

No transfer of energy or mass takes place in the first three phases. However, in the final (fourth) phase transition, transformation of light electronic matter to heavy protonic matter requires transport of energy from the background photons to the proton-building quarks. This is where the Higgs-neutrino or H–Z^0–$\phi^0\phi^+\phi^-\nu\mathrm{W}^+\mathrm{W}^-$ mechanism comes in:

		H		Z		$\phi\nu$W
$\mathrm{e}^+\mathrm{e}^-\ \mathrm{e}^+\mathrm{e}^- \rightarrow$	$\underline{\pi}^+\underline{\pi}^-\ \underline{\pi}^+\underline{\pi}^- \rightarrow$	$\pi_-^+\pi_-^-\ \pi_-^+\pi_-^- \rightarrow$	$\pi_-^+\pi_-^- \rightarrow$	$\pi_+^+\pi_-^- \rightarrow$	$\pi_+^+\pi_+^- \longrightarrow$	$\mathrm{p\bar{p}} \rightarrow \mathrm{pe}^-$
	0	10^{-5}	10^{-5}	10^3	10^{-5}	10^{-5}

Again, the numbers indicate time duration, suggesting that the process takes about $1000\,t_c$ or 10^{-16} seconds.

After the last electron pairs have transformed into equally heavy "frozen" pion pairs (underlined), the mass (or rest energy—no kinetic energy exists) needed to turn the pions into dynamically interacting physical particles is brought by the Higgs. That is, the Higgs is forced to appear on the scene, extract mass from virtual leptons appearing in the propagators of the background photons, and hand it over to u and d quarks forming four massive real pions.

Within less than 10^{-5} time units t_c (with $t_c \approx 10^{-19}$ s) from its creation, one of the pion pairs annihilates via strong interaction.

After a lapse of another $10^{-5}\,t_c$, the imminent annihilation of the remaining pion pair is prevented by the neutral Z boson coming to the rescue—switching the intrinsic parity (indicated by subscript: $\pi_-^+ \rightarrow \pi_+^+$) of one of the pions, thereby making strong decay of the pair impossible. However, in about $10^3\,t_c$, the weak parity-switching force introduced by the Z causes a second change of pion intrinsic parity, which again enables strong decay of the pion pair.

Interestingly, the comparatively long time that elapses between the two parity-switching events introduces a particle-antiparticle asymmetry (point 4.27 on page 60 in Paper.pdf) that should reveal itself in various ways as a kind of "superweak" effect. Presumably, the CP-violating "superweak force" acting in kaon and B-meson decay is a consequence of this matter-antimatter asymmetry.

With the universe's pionic matter doomed to disappear, protonic matter is its natural replacement. Forced by the law of conservation of energy, the Higgs triplets appear on the scene to deliver three times as much mass to the quarks as the Higgs had originally delivered.

Part of this mass remains unused by the quarks and has to be restored to the leptons—the natural alternative in an indeterminate quantum universe where kinetic energy does not exist. The unused mass is returned by neutrinos which, to be able to deliver the mass to the leptons, require help of yet another new type of particle—the charged W boson.

With all real matter of the universe concentrated in a single proton-antiproton pair, the pair's annihilation is forbidden by the law of conservation of energy. Therefore, the antiproton is forced to decay into an electron—an event that signals the end of the universe's labor pains by creating a viable world containing stable proton-electron matter. The antiproton decay introduces kinetic energy and gives matter (the proton and electron) a high temperature.

6 Photon, Higgs, and neutrino

Since the Higgs and neutrino perform similar tasks (transport mass from leptons to quarks and vice versa), they should share a number of properties. Because the Higgs appears first, it should be the simpler of the two particles. Thus, it has no spin while the more complex neutrino possesses spin. Both particles should come in different mass states depending on which lepton (electron, muon, or tauon) emits them. But being simpler than the neutrino, the Higgs should not be able to oscillate between its mass states. For instance, a Higgs emitted by a muon should interact with muons but not with electrons or tauons.

A comparison between photon and Higgs might shed further light on the Higgs. Immediately after the birth of the pion, the photon and the Higgs are the only existing particles capable of interacting with charged leptons. Since they appear in equivalent Feynman diagrams—the Higgs mimicking the photon—one expects the Higgs to resemble the photon. Now, the photon is a massless particle that may vibrate with any frequency and retain any energy that an emitting particle gives it. Similarly, one would expect the Higgs—possibly forming a closed vibrating string—to be capable of carrying any mass it is given by the particle that emits it.

If that is so, the similarity between Higgs-lepton and Higgs-quark vertices suggests that the Higgs comes in as many mass states as there are charged elementary fermions (three leptons and six quarks).

With Higgs mass proportional to the third power of the emitting particle's mass, the mass of the top-type Higgs would be somewhere between 400 and 600 GeV, the bottom-type Higgs would have a mass of roughly 10 MeV, and the rest of the Higgs masses would lie below the electron mass of 0.511 MeV.

The later appearing neutrino is by necessity more complex than the first appearing, maximally simple Higgs. Still, the two particles should resemble each other because of the aforementioned mass-transport role performed by both of them during the final "electronic-to-protonic" phase transition. Thus, if the Higgs is comparable to a closed, non-spinning vibrating string, the neutrino might be looked upon as a closed string that vibrates with three superposed frequencies forming traveling waves that make the neutrino's vibrational energy oscillate and give the particle its spin.

7 Hot versus cold beginning — the Higgs mechanism

It is generally believed that the elementary particles were born massless in the "infinite" heat of the big bang, and that there was perfect symmetry between the electromagnetic and weak forces and between the particles (γ, W^+, W^-, and Z^0) that carry them. The cooling of the immensely hot universe is thought to have triggered a symmetry breaking that—via the Higgs mechanism—gave particles their present masses. This theory predicts the existence of the W and Z particles and leads to a logically consistent model for weak interactions.

However, the Higgs mechanism does not directly predict the value of any specific physical quantity. It may be used to derive the relations

$$M_W^2 = \pi\alpha(\hbar c)^3/\sqrt{2}G_F c^4 \sin^2\theta \tag{7.1}$$

and

$$M_W/M_Z = \cos\theta, \tag{7.2}$$

but the values $M_Z \approx 91$ GeV/c^2 and $M_W \approx 80$ GeV/c^2 are only obtained after the Fermi constant G_F and the Weinberg angle θ have been experimentally determined.

A convincing mechanism should theoretically predict G_F and θ. Also, it should explain the purpose of the weak interactions—why the Z and W bosons and the neutrino (neutral lepton) exist in the first place. The Higgs-neutrino mechanism acting in an originally cold universe explains this.

Therefore, the success of the "hot" Higgs mechanism cannot be taken as proof that the universe originally was immensely hot. What it seems to prove is that SM is a consistent model that—in theory—is valid up to extremely high—in practice unattainable—energies where forces of different strengths converge into a single force.

8 Corrections

In Paper.pdf, the minus sign appearing in Eqs. (9.1), (C.2), and (C.4) should be changed to plus, and one should explain why the actual change of mass of the lepton is negative: At the same time as the first Higgs particle is split off from the lepton causing a negative correction to the lepton mass, the appearance of a virtual Higgs in the lepton propagator causes the lepton's original, purely electromagnetic mass to split into an electromagnetic component and a weak Higgs component.

Similarly, the sign should be changed in Eq. (1) in Experiments.pdf and in Eq. (1) in Draft.pdf.

Also, the details of the computer simulation in Simulation.for and the calculation on page 19 in Experiments.pdf need to be checked. Still, possible corrections should be too small to be of practical significance.

A Calculation of mass correction

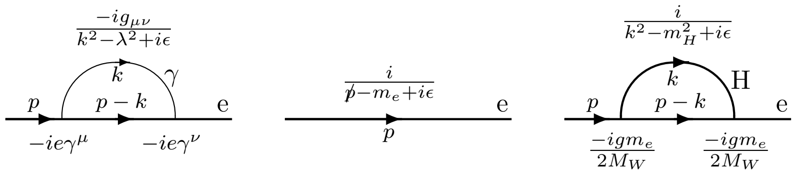
The expression for the second-order photon and Higgs contributions to the lepton mass,

$$\delta m^{(2)}(\gamma) + \delta m^{(2)}(\mathrm{H}) = \ln\frac{\Lambda}{m}\left(\frac{3\alpha}{2\pi} - \frac{3G_F m^2}{8\sqrt{2}\pi^2}\right)m, \tag{A.1}$$

obtained in Paper.pdf (Appendix C on page 33) was derived in an indirect way that requires knowledge of the Feynman-parametric formulation of QED, which was developed by Toichiro

Kinoshita and Predag Cvitanović in 1974 and is presented by Kinoshita on pages 218-321 in Ref [1]. To understand how Eq. (A.1) may be obtained from first principles—that is, from the Feynman rules of SM—one must take a look at these rules.

In the figure, the propagators for the photon, electron, and Higgs are shown above their corresponding particle lines, while the expressions for the photon-electron and Higgs-electron vertices are shown below the electron line:



The notation follows the convention established by James Bjorken and Sidney Drell in their book *Relativistic Quantum Mechanics* [5]—the first of their two standard-setting text books on quantum field theory (QFT) published in 1964 and 1965, respectively.

Thus, in Feynman's slash notation, $\not p$ is the inner product of the four vector γ and the four momentum p, or

$$\not p = \gamma \cdot p = \gamma^\mu p_\mu = \gamma_\mu p^\mu, \tag{A.2}$$

where the convention of summing over repeated indices is used (e.g., $\gamma^\mu p_\mu = \gamma^0 p_0 + \gamma^1 p_1 + \gamma^2 p_2 + \gamma^3 p_3$). For the time component of a four vector such as p, it holds that $p^0 = p_0$, and for its space components, $p^i = -p_i$ ($i = 1,\ 2,\ 3$). The components p_1, p_2, and p_3 form the momentum vector $\boldsymbol{p}$. The same rules apply to the four vector γ ($\gamma^0 = \gamma_0$ and $\gamma^i = -\gamma_i$ with $(\gamma_1,\ \gamma_2,\ \gamma_3) = \boldsymbol{\gamma}$),

The arrows shown in the figure indicate four momentum—p for the electron, and k for the photon and Higgs. The indices μ and ν indicate that summation over photon and electron polarizations must be performed for the photon-electron loop, while no similar summation is needed for the Higgs-electron loop (the reason for the difference being that the photon is a spin-1 boson and the Higgs a spin-0 boson). For computational reasons, the photon is attributed an infinitesimal mass (λ) that is set equal to zero in final results.

Moving clockwise around the loops and multiplying the expressions with each other, one obtains for the integrand associated with the left (photon-electron) loop,

$$I(\gamma) = \frac{-ig_{\mu\nu}}{k^2 - \lambda^2 + i\epsilon} \left(-ie\gamma^\nu\right) \frac{i}{\not p - \not k - m_e + i\epsilon} \left(-ie\gamma^\mu\right), \tag{A.3}$$

and for the integrand associated with the right (Higgs-electron) loop,

$$I(\mathrm{H}) = \frac{i}{k^2 - m_H^2 + i\epsilon} \left(-ig\frac{m_e}{2M_W}\right) \frac{i}{\not p - \not k - m_e + i\epsilon} \left(-ig\frac{m_e}{2M_W}\right). \tag{A.4}$$

The symbol $g_{\mu\nu}$ appearing in the photon propagator is given by the 4×4 matrix (p. 281 in Ref. [5])

$$g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix},$$

$$\gamma^0 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}, \ \gamma^1 = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{bmatrix}, \ \gamma^2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix}, \ \gamma^3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix}.$$

where only the nonzero elements of the matrix are explicitly shown. Similarly, the components of the four vector γ are the Dirac matrices (p. 282)
The fundamental property of the γ matrices is the anticommutation relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}, \tag{A.5}$$

from which the rest of their properties may be derived using the fact that $g_{\mu\nu}$ lowers the index of a four-vector component while $g^{\mu\nu}$ raises it;

$$g_{\mu\nu}\gamma^\nu = \gamma_\mu, \quad g^{\mu\nu}\gamma_\nu = \gamma^\mu, \quad g_{\mu\nu}p^\nu = p_\mu, \quad g^{\mu\nu}p_\nu = p^\mu. \tag{A.6}$$

For instance, multiplication of Eq. (A.5) by $p_\mu q_\nu$ yields

$$\not{p}\not{q} + \not{q}\not{p} = 2p_\mu q^\mu = 2p \cdot q \tag{A.7}$$

(since, being scalar quantities, p_μ and q_ν commute with γ matrices; $\gamma^\nu p^\mu = p^\mu \gamma^\nu$). With $q = p$, the relation simplifies to

$$\not{p}^2 = p^2. \tag{A.8}$$

Also, readily obtained are the relations

$$\gamma_\mu \gamma^\mu = 4, \quad \gamma_\mu \not{p} \gamma^\mu = -2\not{p}, \tag{A.9}$$

the latter via $\gamma_\mu \not{p} \gamma^\mu = \gamma_\mu \gamma_\alpha p^\alpha \gamma^\mu = (2g_{\mu\alpha} - \gamma_\alpha \gamma_\mu)\gamma^\mu p^\alpha = (2\gamma_\alpha - \gamma_\alpha \gamma_\mu \gamma^\mu)p^\alpha = -2\gamma_\alpha p^\alpha$.

Ignoring the infinitesimal constant ϵ, using $g_{\mu\nu}\gamma^\nu = \gamma_\mu$, and introducing the fine-structure constant α and the Fermi coupling constant G_F via the relations

$$e^2 = 4\pi\alpha, \quad G_F/\sqrt{2} = g^2/8M_W^2 \tag{A.10}$$

(see p. 159 in Ref. [6]), the integrands may be written

$$I(\gamma) = -4\pi\alpha \frac{\gamma_\mu(\not{p} - \not{k} + m_e)\gamma^\mu}{(k^2 - \lambda^2)((p-k)^2 - m_e^2)} \tag{A.11}$$

and

$$I(\mathrm{H}) = \sqrt{2}G_F m_e^2 \frac{\not{p} - \not{k} + m_e}{(k^2 - m_H^2)((p-k)^2 - m_e^2)} \tag{A.12}$$

when the electron propagator is rewritten according to

$$\frac{1}{\not{p} - m_e} = \frac{1}{\not{p} - m_e} \times \frac{\not{p} + m_e}{\not{p} + m_e} = \frac{\not{p} + m_e}{\not{p}^2 - m_e^2} = \frac{\not{p} + m_e}{p^2 - m_e^2}. \tag{A.13}$$

Before the integrands can be weighed against each other, the numerator in Eq. (A.11) must be simplified. With the aid of Eq. (A.9), the integrand becomes

$$I(\gamma) = 8\pi\alpha \frac{\not{p} - \not{k} - 2m_e}{(k^2 - \lambda^2)((p-k)^2 - m_e^2)}. \tag{A.14}$$

Integration over the four momentum k produces a divergent result for k approaching infinity—hence the UV cutoff mass Λ in Eq. (1.3). The fact that m_H^2 and m_e^2 appear alongside k^2, and m_e alongside k, explains why no particle masses appear in the divergent part of the expression for $\delta m^{(2)}$ (since $m_e/\gamma k$, m_e^2/k^2, and $m_H^2/k^2 \to 0$ for $k \to \infty$).

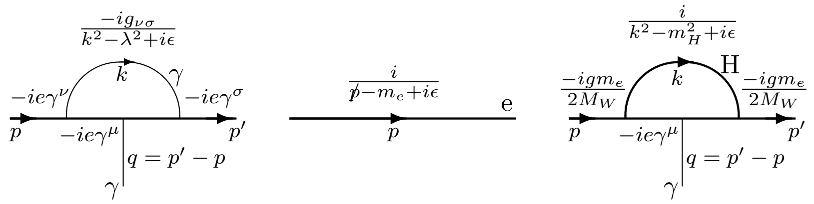
Division of Eq. (A.12) by Eq. (A.14) shows that in the limit when $k \to \infty$ (and the integral diverges), the ratio between the two integrands is

$$\frac{I(\mathrm{H})}{I(\gamma)} = \frac{G_F m_e^2}{4\sqrt{2}\pi\alpha}, \tag{A.15}$$

which is the same ratio as in Eq. (A.1), but with opposite sign. Consequently, the result obtained in Eq. (A.15) demonstrates a mistake in the original calculation.

B Higgs contribution to $g - 2$

On pages 166–172 in their text book [5], Bjorken and Drell calculate the part, $a_e^{(2)}(\gamma)$, of the electron anomalous magnetic moment that derives from the figure's left diagram:



The diagram consists of a self-mass loop with an external photon line added. The photon mediates the force between the external magnetic field and the electron. Its momentum q is the difference between the electron's final momentum p' and its initial momentum p.

The expression obtained from the Feynman rules may be written as $-ie\Lambda_\mu(p', p)$, where $\Lambda_\mu(p', p)$ is given by

$$\begin{aligned}\Lambda_\mu(p', p) &= \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{-ig_{\nu\sigma}}{k^2 - \lambda^2 + i\epsilon} (-ie\gamma^\sigma) \frac{i}{\not p' - \not k - m_e + i\epsilon} \gamma_\mu \frac{i}{\not p - \not k - m_e + i\epsilon} (-ie\gamma^\nu) \\ &= (-ie)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{-i}{k^2 - \lambda^2 + i\epsilon} \gamma_\nu \frac{i}{\not p' - \not k - m_e + i\epsilon} \gamma_\mu \frac{i}{\not p - \not k - m_e + i\epsilon} \gamma^\nu \\ &= (-ie)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i}{k^2 - \lambda^2} \frac{\gamma_\nu (\not p' - \not k + m_e) \gamma_\mu (\not p - \not k + m_e) \gamma^\nu}{((p' - k)^2 - m_e^2)((p - k)^2 - m_e^2)}.\end{aligned} \tag{B.1}$$

See Eq. (8.49) on page 166 in the text book. For the corresponding Higgs contribution (right diagram), one similarly obtains

$$\begin{aligned}\Lambda_\mu^{\mathrm{H}}(p', p) &= \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i}{k^2 - m_H^2 + i\epsilon} \frac{-igm_e}{2M_W} \frac{i}{\not p' - \not k - m_e + i\epsilon} \gamma_\mu \frac{i}{\not p - \not k - m_e + i\epsilon} \frac{-igm_e}{2M_W} \\ &= \left(\frac{-igm_e}{2M_W}\right)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{-i}{k^2 - m_H^2} \frac{(\not p' - \not k + m_e) \gamma_\mu (\not p - \not k + m_e)}{((p' - k)^2 - m_e^2)((p - k)^2 - m_e^2)}.\end{aligned} \tag{B.2}$$

Because of the similarity between the expressions, the Higgs contribution may be found via a parallel calculation of the two cases. The calculation is a mess.[1] However, only a small part of it has to be redone for the Higgs.

In Eqs. (8.49) and (8.60) it is enough to simply ignore the γ_ν and γ^ν matrices. Thus, the respective numerator (n) of the last term in Eq. (8.60) is

$$\begin{aligned} n(\gamma) &= \gamma_\nu[\not{p}'(1-z_2) - \not{p}z_3 + m]\gamma_\mu[\not{p}(1-z_3) - \not{p}'z_2 + m]\gamma^\nu, \\ n(\mathrm{H}) &= [\not{p}'(1-z_2) - \not{p}z_3 + m]\gamma_\mu[\not{p}(1-z_3) - \not{p}'z_2 + m] \end{aligned} \tag{B.3}$$

for the two cases.

In the Higgs case, evaluation of the numerator (consisting in shifting $\not{p}'$ left and $\not{p}$ right) yields

$$\begin{aligned} n(\mathrm{H}) = {} & z_1\not{p}'\gamma_\mu\not{p} \\ & + \not{p}'[m\gamma_\mu + 2z_2z_3p_\mu - 2z_2(1-z_2)p'_\mu] \\ & + [m\gamma_\mu + 2z_2z_3p'_\mu - 2z_3(1-z_3)p_\mu]\not{p} \\ & + [-2z_2z_3p\cdot p' + z_3(1-z_3)p^2 + z_2(1-z_2)p'^2 + m^2]\gamma_\mu \\ & - 2mz_3p_\mu - 2mz_2p'_\mu. \end{aligned} \tag{B.4}$$

The result is obtained via repeated application of Eq. (A.5). A safe way to verify the steps of the calculation is to give randomly chosen numerical values to the variables (with restrictions $z_1 + z_2 + z_3 = 1$ and $p^2 = p'^2 = m^2$) and check on a computer that the consecutively obtained expressions yield numerically identical 4×4 complex matrices.

Noting that $q^2 = p'^2 - 2p'\cdot p + p^2$, using $p^2 = p'^2 = m^2$, and letting $\not{p} \to m$ and $\not{p}' \to m$ (see Eq. (3.9a) on p. 30 in *Relativistic Quantum Mechanics*), the numerators simplify to

$$\begin{aligned} n(\gamma) &= -\gamma_\mu[2m^2(1-2z_1-z_1^2) + 2q^2(1-z_2)(1-z_3)] \\ &\quad + 4m[(z_1z_2 - z_2 + z_3)p'_\mu + (z_1z_3 - z_3 + z_2)p_\mu], \\ n(\mathrm{H}) &= \gamma_\mu[m^2(3+2z_1-z_1^2) + q^2z_2z_3] \\ &\quad - 2m[(1+z_1)z_2p'_\mu + (1+z_1)z_3p_\mu]. \end{aligned} \tag{B.5}$$

Because z_2 and z_3 are interchangeable in the integration and the denominator is symmetric under $z_2 \leftrightarrow z_3$, the p'_μ and p_μ terms may be combined into $-4m(z_1z_2 - z_2 + z_3)(p'_\mu + p_\mu)$ and $-2m(1+z_1)z_2(p'_\mu + p_\mu)$, respectively. Application of Gordon decomposition ($(p'+p)_\mu \to 2m\gamma_\mu - \frac{1}{2}(\not{q}\gamma_\mu - \gamma_\mu\not{q}) = 2m\gamma_\mu - \frac{1}{2}[\not{q}, \gamma_\mu]$) results in

$$\begin{aligned} n(\gamma) &= -2\gamma_\mu m^2[(1-2z_1-z_1^2 - 4(z_1z_2 - z_2 + z_3)] - 2\gamma_\mu q^2(1-z_2)(1-z_3) \\ &\quad - 2m(z_1z_2 - z_2 + z_3)[\not{q}, \gamma_\mu], \\ n(\mathrm{H}) &= \gamma_\mu m^2[(3 + 2z1 - z1^2 - 4(1+z_1)z_2] + \gamma_\mu q^2 z_2z_3 + m(1+z_1)z_2[\not{q}, \gamma_\mu]. \end{aligned} \tag{B.6}$$

Ignoring $-z_2 + z_3$ (which, being odd under interchange $x_2 \leftrightarrow x_3$, contributes zero to the integral) and replacing $2z_2$ by $z_2 + z_3 = 1 - z_1$, one obtains for the numerators

$$\begin{aligned} n(\gamma) &= -\gamma_\mu[2m^2(1-4z_1+z_1^2) + 2q^2(1-z_2)(1-z_3)] - 2mz_1z_2[\not{q}, \gamma_\mu], \\ n(\mathrm{H}) &= \gamma_\mu[m^2(1+z1)^2 + q^2z_2z_3] + m(1+z_1)z_2[\not{q}, \gamma_\mu]. \end{aligned} \tag{B.7}$$

[1] An easy-to-follow calculation of $a_e^{(2)}(\gamma)$ is found in Example 5.6 on page 272 in W. Greiner, J. Reinhardt, *Quantum Electrodynamics*, 2nd edition, Springer, Berlin, 1996.

In the limit when $q^2,\ \lambda^2,\ m_H^2 \to 0$, integration of the last terms in Eq. (B.7) yields

$$\begin{aligned} &-\frac{4\pi\alpha}{16\pi^2} \times \frac{-2m[\not{q}, \gamma\mu]}{m^2} \int\limits_0^1 \mathrm{d}z_1 \int\limits_0^{1-z_1} \mathrm{d}z_2 \frac{z_1 z_2}{(1-z_1)^2} = \frac{\alpha}{8\pi m}[\not{q}, \gamma\mu], \\ &\frac{\sqrt{2}G_F m^2}{16\pi^2} \times \frac{m[\not{q}, \gamma\mu]}{m^2} \int\limits_0^1 \mathrm{d}z_1 \int\limits_0^{1-z_1} \mathrm{d}z_2 \frac{(1+z_1)z_2}{(1-z_1)^2} = \frac{3\sqrt{2}G_F m}{64\pi^2}[\not{q}, \gamma\mu]. \end{aligned} \tag{B.8}$$

A comparison between the two expressions gives for the respective correction

$$\begin{aligned} a_e^{(2)}(\gamma) &= \frac{\alpha}{2\pi}, \\ a_e^{(2)}(\mathrm{H}) &= \frac{3\sqrt{2}G_F m^2}{16\pi^2} = \frac{3G_F m^2}{8\sqrt{2}\pi^2}, \end{aligned} \tag{B.9}$$

where the correction to a_e is given in Eq. (8.64) in the text book.

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